65. (a) Since the line passes through the origin and has slope $\frac{1}{e}$, its equation is $y = \frac{x}{e}$.

(b) The graph of $y = \ln x$ lies below the graph of the line $y = \frac{x}{e}$ for all positive $x \neq e$. Therefore, $\ln x < \frac{x}{e}$ for all positive $x \neq e$.

(c) Multiplying by e, $e \ln x < x$ or $\ln x^e < x$.

(d) Exponentiating both sides of $\ln x^e < x$, we have $e^{\ln x^e} < e^x$, or $x^e < e^x$ for all positive $x \neq e$.

(e) Let $x = \pi$ to see that $\pi^e < e^\pi$. Therefore, e^π is bigger.

Quick Quiz Sections 3.7–3.9

1. E. $y = \frac{9}{2x} - \frac{x^2}{2}$

$$\frac{dy}{dx} = -\frac{9}{2x^2} - x$$

$$\left.\frac{dy}{dx}\right|_{x=1} = -\frac{9}{2(1)^2} - 1 = -\frac{11}{2}$$

2. A. $\frac{dy}{dx} = \frac{d}{dx}(\cos(3x-2))^3$

$$= 3(\cos(3x-2))^2(-\sin(3x-2))(3)$$

$$= -9\cos^2(3x-2)\sin(3x-2)$$

3. C. $\frac{dy}{dx} = \frac{d}{dx}(\sin^{-1}(2x))$

$$= \frac{1}{\sqrt{1-(2x)^2}}\frac{d}{dx}(2x)$$

$$= \frac{2}{\sqrt{1-4x^2}}$$

4. (a) Differentiate implicitly:

$$\frac{d}{dx}(xy^2 - x^3y) = \frac{d}{dx}(6)$$

$$1 \cdot y^2 + x \cdot 2y\frac{dy}{dx} - \left(3x^2y + x^3\frac{dy}{dx}\right) = 0$$

$$2xy\frac{dy}{dx} - x^3\frac{dy}{dx} = 3x^2y - y^2$$

$$\frac{dy}{dx} = \frac{3x^2y - y^2}{2xy - x^3}$$

(b) If $x = 1$, then $y^2 - y = 6$, so $y = -2$ or $y = 3$.

at $(1, -2)$, $\frac{dy}{dx} = \frac{3(1)^2(-2)-(-2)^2}{2(1)(-2)-(1)^3} = 2.$

The tangent line is $y + 2 = 2(x - 1)$.

At $(1, 3)$, $\frac{dy}{dx} = \frac{3(1)^2(3)-3^2}{2(1)(3)-1^3} = 0.$

The tangent line is $y = 3$.

(c) The tangent line is vertical where $2xy - x^3 = 0$, which implies $x = 0$ or $y = \frac{x^2}{2}$. There is no point on the curve where $x = 0$. If $y = \frac{x^2}{2}$, then $x\left(\frac{x^2}{2}\right)^2 - x^3\left(\frac{x^2}{2}\right) = 6.$

Then the only solution to this equation is $x = \sqrt[5]{-24}$.

Chapter 3 Review Exercises

(pp. 181–184)

1. $\frac{dy}{dx} = \frac{d}{dx}\left(x^5 - \frac{1}{8}x^2 + \frac{1}{4}x\right) = 5x^4 - \frac{1}{4}x + \frac{1}{4}$

2. $\frac{dy}{dx} = \frac{d}{dx}(3 - 7x^3 + 3x^7) = -21x^2 + 21x^6$

3. $\frac{dy}{dx} = \frac{d}{dx}(2\sin x\cos x)$

$$= 2(\sin x)\frac{d}{dx}(\cos x) + 2(\cos x)\frac{d}{dx}(\sin x)$$

$$= -2\sin^2 x + 2\cos^2 x$$

Alternate solution:

$$\frac{dy}{dx} = \frac{d}{dx}(2\sin x\cos x) = \frac{d}{dx}\sin 2x = (\cos 2x)(2)$$

$$= 2\cos 2x$$

4. $\frac{dy}{dx} = \frac{d}{dx}\left(\frac{2x+1}{2x-1}\right) = \frac{(2x-1)(2)-(2x+1)(2)}{(2x-1)^2} = -\frac{4}{(2x-1)^2}$

5. $\frac{ds}{dt} = \frac{d}{dt}\cos(1-2t) = -\sin(1-2t)(-2) = 2\sin(1-2t)$

6. $\frac{ds}{dt} = \frac{d}{dt}\cot\left(\frac{2}{t}\right) = -\csc^2\left(\frac{2}{t}\right)\frac{d}{dt}\left(\frac{2}{t}\right) = -\csc^2\left(\frac{2}{t}\right)\left(-\frac{2}{t^2}\right)$

$$= \frac{2}{t^2}\csc^2\left(\frac{2}{t}\right)$$

7. $\frac{dy}{dx} = \frac{d}{dx}\left(\sqrt{x} + 1 + \frac{1}{\sqrt{x}}\right) = \frac{d}{dx}(x^{1/2} + 1 + x^{-1/2})$

$$= \frac{1}{2}x^{-1/2} - \frac{1}{2}x^{-3/2} = \frac{1}{2\sqrt{x}} - \frac{1}{2x^{3/2}}$$

8. $\frac{dy}{dx} = \frac{d}{dx}(x\sqrt{2x+1}) = (x)\left(\frac{1}{2\sqrt{2x+1}}\right)(2) + (\sqrt{2x+1})(1)$

$$= \frac{x + (2x+1)}{\sqrt{2x+1}} = \frac{3x+1}{\sqrt{2x+1}}$$

9. $\frac{dr}{d\theta} = \frac{d}{d\theta}\sec(1+3\theta) = \sec(1+3\theta)\tan(1+3\theta)(3)$

$$= 3\sec(1+3\theta)\tan(1+3\theta)$$

10. $\frac{dr}{d\theta} = \frac{d}{d\theta}\tan^2(3-\theta^2)$

$$= 2\tan(3-\theta^2)\frac{d}{d\theta}\tan(3-\theta^2)$$

$$= 2\tan(3-\theta^2)\sec^2(3-\theta^2)(-2\theta)$$

$$= -4\theta\tan(3-\theta^2)\sec^2(3-\theta^2)$$

11. $\frac{dy}{dx} = \frac{d}{dx}(x^2 \csc 5x)$

$= (x^2)(-\csc 5x \cot 5x)(5) + (\csc 5x)(2x)$

$= -5x^2 \csc 5x \cot 5x + 2x \csc 5x$

12. $\frac{dy}{dx} = \frac{d}{dx}\ln\sqrt{x} = \frac{1}{\sqrt{x}}\frac{d}{dx}\sqrt{x} = \frac{1}{\sqrt{x}} \cdot \frac{1}{2\sqrt{x}} = \frac{1}{2x}, x > 0$

13. $\frac{dy}{dx} = \frac{d}{dx}\ln(1+e^x) = \frac{1}{1+e^x}\frac{d}{dx}(1+e^x) = \frac{e^x}{1+e^x}$

14. $\frac{dy}{dx} = \frac{d}{dx}(xe^{-x}) = (x)(e^{-x})(-1) + (e^{-x})(1) = -xe^{-x} + e^{-x}$

15. $\frac{dy}{dx} = \frac{d}{dx}(e^{1+\ln x}) = \frac{d}{dx}(e^1 e^{\ln x}) = \frac{d}{dx}(ex) = e$

16. $\frac{dy}{dx} = \frac{d}{dx}\ln(\sin x) = \frac{1}{\sin x}\frac{d}{dx}(\sin x) = \frac{\cos x}{\sin x} = \cot x$, for values of x in the intervals $(k\pi, (k+1)\pi)$, where k is even.

17. $\frac{dr}{dx} = \frac{d}{dx}\ln(\cos^{-1} x) = \frac{1}{\cos^{-1} x}\frac{d}{dx}\cos^{-1} x$

$= \frac{1}{\cos^{-1} x}\left(-\frac{1}{\sqrt{1-x^2}}\right) = -\frac{1}{\cos^{-1} x\sqrt{1-x^2}}$ for $-1 < x < 1$

18. $\frac{dr}{d\theta} = \frac{d}{d\theta}\log_2(\theta^2) = \frac{1}{\theta^2 \ln 2}\frac{d}{d\theta}(\theta^2) = \frac{2\theta}{\theta^2 \ln 2} = \frac{2}{\theta \ln 2}$

19. $\frac{ds}{dt} = \frac{d}{dt}\log_5(t-7) = \frac{1}{(t-7)\ln 5}\frac{d}{dt}(t-7) = \frac{1}{(t-7)\ln 5}$, $t > 7$

20. $\frac{ds}{dt} = \frac{d}{dt}(8^{-t}) = 8^{-t}(\ln 8)\frac{d}{dt}(-t) = -8^{-t}\ln 8$

21. Use logarithmic differentiation.

$y = x^{\ln x}$

$\ln y = \ln(x^{\ln x})$

$\ln y = (\ln x)(\ln x)$

$\frac{d}{dx}\ln y = \frac{d}{dx}(\ln x)^2$

$\frac{1}{y}\frac{dy}{dx} = 2\ln x\frac{d}{dx}\ln x$

$\frac{dy}{dx} = \frac{2y\ln x}{x}$

$\frac{dy}{dx} = \frac{2x^{\ln x}\ln x}{x}$

22. $\frac{dy}{dx} = \frac{d}{dx}\frac{(2x)2^x}{\sqrt{x^2+1}}$

$= \frac{\sqrt{x^2+1}\frac{d}{dx}\left[(2x)2^x\right] - (2x)(2^x)\frac{d}{dx}\sqrt{x^2+1}}{x^2+1}$

$= \frac{\sqrt{x^2+1}\left[(2x)(2^x)(\ln 2) + (2^x)(2)\right] - (2x)(2^x)\frac{1}{2\sqrt{x^2+1}}(2x)}{x^2+1}$

$= \frac{(x^2+1)(2^x)(2x\ln 2 + 2) - 2x^2(2^x)}{(x^2+1)^{3/2}}$

$= \frac{(2 \cdot 2^x)[(x^2+1)(x\ln 2 + 1) - x^2]}{(x^2+1)^{3/2}}$

$= \frac{(2 \cdot 2^x)(x^3\ln 2 + x^2 + x\ln 2 + 1 - x^2)}{(x^2+1)^{3/2}}$

$= \frac{(2 \cdot 2^x)(x^3\ln 2 + x\ln 2 + 1)}{(x^2+1)^{3/2}}$

Alternate solution, using logarithmic differentiation:

$y = \frac{(2x)2^x}{\sqrt{x^2+1}}$

$\ln y = \ln(2x) + \ln(2^x) - \ln\sqrt{x^2+1}$

$\ln y = \ln 2 + \ln x + x\ln 2 - \frac{1}{2}\ln(x^2+1)$

$\frac{d}{dx}\ln y = \frac{d}{dx}[\ln 2 + \ln x + x\ln 2 - \frac{1}{2}\ln(x^2+1)]$

$\frac{1}{y}\frac{dy}{dx} = 0 + \frac{1}{x} + \ln 2 - \frac{1}{2}\frac{1}{x^2+1}(2x)$

$\frac{dy}{dx} = y\left(\frac{1}{x} + \ln 2 - \frac{x}{x^2+1}\right)$

$\frac{dy}{dx} = \frac{(2x)2^x}{\sqrt{x^2+1}}\left(\frac{1}{x} + \ln 2 - \frac{x}{x^2+1}\right)$

23. $\frac{dy}{dx} = \frac{d}{dx}e^{\tan^{-1}x} = e^{\tan^{-1}x}\frac{d}{dx}\tan^{-1}x = \frac{e^{\tan^{-1}x}}{1+x^2}$

24. $\frac{dy}{du} = \frac{d}{du}\sin^{-1}\sqrt{1-u^2}$

$= \frac{1}{\sqrt{1-(\sqrt{1-u^2})^2}}\frac{d}{du}\sqrt{1-u^2}$

$= \frac{1}{\sqrt{u^2}}\frac{1}{2\sqrt{1-u^2}}(-2u) = \frac{-u}{|u|\sqrt{1-u^2}}$

25. $\frac{dy}{dt} = \frac{d}{dt}\left(t\sec^{-1}t - \frac{1}{2}\ln t\right)$

$= (t)\left(\frac{1}{|t|\sqrt{t^2-1}}\right) + (\sec^{-1}t)(1) - \frac{1}{2t}$

$= \frac{t}{|t|\sqrt{t^2-1}} + \sec^{-1}t - \frac{1}{2t}$

26. $\frac{dy}{dt} = \frac{d}{dt}[(1+t^2)\cot^{-1}2t]$

$= (1+t^2)\left(-\frac{1}{1+(2t)^2}\right)(2) + (\cot^{-1}2t)(2t)$

$= -\frac{2+2t^2}{1+4t^2} + 2t\cot^{-1}2t$

27. $\frac{dy}{dz}=\frac{d}{dz}(z\cos^{-1}z-\sqrt{1-z^2})$

$=(z)\left(-\frac{1}{\sqrt{1-z^2}}\right)+(\cos^{-1}z)(1)-\frac{1}{2\sqrt{1-z^2}}(-2z)$

$=-\frac{z}{\sqrt{1-z^2}}+\cos^{-1}z+\frac{z}{\sqrt{1-z^2}}$

$=\cos^{-1}z$

28. $\frac{dy}{dx}=\frac{d}{dx}(2\sqrt{x-1}\csc^{-1}\sqrt{x})$

$=(2\sqrt{x-1})\left(-\frac{1}{|\sqrt{x}|\sqrt{(\sqrt{x})^2-1}}\right)\left(\frac{1}{2\sqrt{x}}\right)$

$+(2\csc^{-1}\sqrt{x})\left(\frac{1}{2\sqrt{x-1}}\right)$

$=-\frac{\sqrt{x-1}}{(\sqrt{x})^2\sqrt{x-1}}+\frac{\csc^{-1}\sqrt{x}}{\sqrt{x-1}}$

$=-\frac{1}{x}+\frac{\csc^{-1}\sqrt{x}}{\sqrt{x-1}}$

29. $\frac{dy}{dx}=\frac{d}{dx}\csc^{-1}(\sec x)$

$=\left(-\frac{1}{|\sec x|\sqrt{\sec^2 x-1}}\right)\frac{d}{dx}(\sec x)$

$=-\frac{1}{|\sec x|\sqrt{\tan^2 x}}\sec x\tan x$

$=-\frac{\sec x\tan x}{|\sec x\tan x|}$

$=-\frac{\frac{1}{\cos x}\frac{\sin x}{\cos x}}{\left|\frac{1}{\cos x}\frac{\sin x}{\cos x}\right|}=-\frac{\sin x}{|\sin x|}$

$=\begin{cases}-1, & 0\le x<\pi, \quad x\ne\frac{\pi}{2}\\ 1, & \pi<x\le 2\pi, \quad x\ne\frac{3\pi}{2}\end{cases}$

Alternate method:

On the domain $0\le x\le 2\pi$, $x\ne\frac{\pi}{2}$, $x\ne\frac{3\pi}{2}$, we may rewrite the function as follows:

$y=\csc^{-1}(\sec x)$

$=\frac{\pi}{2}-\sec^{-1}(\sec x)$

$=\frac{\pi}{2}-\cos^{-1}(\cos x)$

$=\begin{cases}\frac{\pi}{2}-x, & 0\le x\le\pi, \quad x\ne\frac{\pi}{2}\\ \frac{\pi}{2}-(\pi-x), & \pi<x\le 2\pi, \quad x\ne\frac{3\pi}{2}\end{cases}$

$=\begin{cases}\frac{\pi}{2}-x, & 0\le x\le\pi, \quad x\ne\frac{\pi}{2}\\ -\frac{\pi}{2}+x, & \pi<x\le 2\pi, \quad x\ne\frac{3\pi}{2}\end{cases}$

Therefore, $\frac{dy}{dx}=\begin{cases}-1, & 0\le x<\pi, \quad x\ne\frac{\pi}{2}\\ 1, & \pi<x\le 2\pi, \quad x\ne\frac{3\pi}{2}\end{cases}$

Note that the derivative exists at 0 and 2π only because these are the endpoints of the given domain; the two-sided derivative of $y=\csc^{-1}(\sec x)$ does not exist at these points.

30. $\frac{dr}{d\theta}=\frac{d}{d\theta}\left(\frac{1+\sin\theta}{1-\cos\theta}\right)^2$

$=2\left(\frac{1+\sin\theta}{1-\cos\theta}\right)\left(\frac{(1-\cos\theta)(\cos\theta)-(1+\sin\theta)(\sin\theta)}{(1-\cos\theta)^2}\right)$

$=2\left(\frac{1+\sin\theta}{1-\cos\theta}\right)\left(\frac{\cos\theta-\cos^2\theta-\sin\theta-\sin^2\theta}{(1-\cos\theta)^2}\right)$

$=2\left(\frac{1+\sin\theta}{1-\cos\theta}\right)\left(\frac{\cos\theta-\sin\theta-1}{(1-\cos\theta)^2}\right)$

31. Since $y=\ln x^2$ is defined for all $x\ne 0$ and $\frac{dy}{dx}=\frac{1}{x^2}\frac{d}{dx}(x^2)=\frac{2x}{x^2}=\frac{2}{x}$, the function is differentiable for all $x\ne 0$.

32. Since $y=\sin x-x\cos x$ is defined for all real x and $\frac{dy}{dx}=\cos x-(x)(-\sin x)-(\cos x)(1)=x\sin x$, the function is differentiable for all real x.

33. Since $y=\sqrt{\frac{1-x}{1+x^2}}$ is defined for all $x<1$ and

$\frac{dy}{dx}=\frac{1}{2}\left(\frac{1-x}{1+x^2}\right)^{-1/2}\frac{(1+x^2)(-1)-(1-x)(2x)}{(1+x^2)^2}$

$=\frac{x^2-2x-1}{2\sqrt{1-x}(1+x^2)^{3/2}}$, which is defined only for $x<1$, the function is differentiable for all $x<1$.

34. Since $y=(2x-7)^{-1}(x+5)=\frac{x+5}{2x-7}$ is defined for all $x\ne\frac{7}{2}$ and $\frac{dy}{dx}=\frac{(2x-7)(1)-(x+5)(2)}{(2x-7)^2}=-\frac{17}{(2x-7)^2}$, the function is differentiable for all $x\ne\frac{7}{2}$.

35. Use implicit differentiation.

$$xy+2x+3y=1$$

$$\frac{d}{dx}(xy)+\frac{d}{dx}(2x)+\frac{d}{dx}(3y)=\frac{d}{dx}(1)$$

$$x\frac{dy}{dx}+(y)(1)+2+3\frac{dy}{dx}=0$$

$$(x+3)\frac{dy}{dx}=-(y+2)$$

$$\frac{dy}{dx}=-\frac{y+2}{x+3}$$

36. Use implicit differentiation.

$$5x^{4/5} + 10y^{6/5} = 15$$

$$\frac{d}{dx}(5x^{4/5}) + \frac{d}{dx}(10y^{6/5}) = \frac{d}{dx}(15)$$

$$4x^{-1/5} + 12y^{1/5}\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = -\frac{4x^{-1/5}}{12y^{1/5}} = -\frac{1}{3(xy)^{1/5}}$$

37. Use implicit differentiation.

$$\sqrt{xy} = 1$$

$$\frac{d}{dx}\sqrt{xy} = \frac{d}{dx}(1)$$

$$\frac{1}{2\sqrt{xy}}\left[x\frac{dy}{dx} + (y)(1)\right] = 0$$

$$x\frac{dy}{dx} + y = 0$$

$$\frac{dy}{dx} = -\frac{y}{x}$$

Alternate method: $\ln\sqrt{xy} = \ln(1)$

$$\frac{1}{2}[\ln x + \ln y] = 0$$

$$\ln x + \ln y = 0$$

$$\frac{1}{x} + \frac{1}{y}\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = -\frac{y}{x}$$

38. Use implicit differentiation.

$$y^2 = \frac{x}{x+1}$$

$$\frac{d}{dx}y^2 = \frac{d}{dx}\frac{x}{x+1}$$

$$2y\frac{dy}{dx} = \frac{(x+1)(1) - (x)(1)}{(x+1)^2}$$

$$\frac{dy}{dx} = \frac{1}{2y(x+1)^2}$$

39. $x^3 + y^3 = 1$

$$\frac{d}{dx}(x^3) + \frac{d}{dx}(y^3) = \frac{d}{dx}(1)$$

$$3x^2 + 3y^2y' = 0$$

$$y' = -\frac{x^2}{y^2}$$

$$y'' = \frac{d}{dx}\left(-\frac{x^2}{y^2}\right) = -\frac{(y^2)(2x) - (x^2)(2y)(y')}{y^4}$$

$$= -\frac{(y^2)(2x) - (x^2)(2y)\left(-\frac{x^2}{y^2}\right)}{y^4}$$

$$= -\frac{2xy^3 + 2x^4}{y^5} = -\frac{2x(x^3+y^3)}{y^5} = -\frac{2x}{y^5}$$

since $x^3 + y^3 = 1$

40. $y^2 = 1 - \frac{2}{x}$

$$\frac{d}{dx}(y^2) = \frac{d}{dx}(1) - \frac{d}{dx}\left(\frac{2}{x}\right)$$

$$2yy' = \frac{2}{x^2}$$

$$y' = \frac{2}{x^2(2y)} = \frac{1}{x^2y}$$

$$y'' = \frac{d}{dx}\left(\frac{1}{x^2y}\right)$$

$$= -\frac{1}{(x^2y)^2}\frac{d}{dx}(x^2y)$$

$$= -\frac{1}{(x^2y)^2}[(x^2)(y') + (y)(2x)]$$

$$= -\frac{1}{(x^2y)^2}\left[(x^2)\left(\frac{1}{x^2y}\right) + 2xy\right]$$

$$= -\frac{1}{x^4y^2}\left(\frac{1}{y} + 2xy\right)$$

$$= -\frac{1+2xy^2}{x^4y^3}$$

41. $y^3 + y = 2\cos x$

$$\frac{d}{dx}(y^3) + \frac{d}{dx}(y) = \frac{d}{dx}(2\cos x)$$

$$3y^2y' + y' = -2\sin x$$

$$(3y^2+1)y' = -2\sin x$$

$$y' = -\frac{2\sin x}{3y^2+1}$$

$$y'' = \frac{d}{dx}\left(-\frac{2\sin x}{3y^2+1}\right)$$

$$= -\frac{(3y^2+1)(2\cos x) - (2\sin x)(6yy')}{(3y^2+1)^2}$$

$$= -\frac{(3y^2+1)(2\cos x) - (12y\sin x)\left(-\frac{2\sin x}{3y^2+1}\right)}{(3y^2+1)^2}$$

$$= -2\frac{(3y^2+1)^2\cos x + 12y\sin^2 x}{(3y^2+1)^3}$$

42.
$$x^{1/3}+y^{1/3}=4$$
$$\frac{d}{dx}(x^{1/3})+\frac{d}{dx}(y^{1/3})=\frac{d}{dx}(4)$$
$$\frac{1}{3}x^{-2/3}+\frac{1}{3}y^{-2/3}y'=0$$
$$y'=-\frac{x^{-2/3}}{y^{-2/3}}=-\left(\frac{y}{x}\right)^{2/3}$$
$$y''=\frac{d}{dx}\left[-\left(\frac{y}{x}\right)^{2/3}\right]$$
$$=-\frac{2}{3}\left(\frac{y}{x}\right)^{-1/3}\left(\frac{xy'-(y)(1)}{x^2}\right)$$
$$=-\frac{2}{3}\left(\frac{y}{x}\right)^{-1/3}\left(\frac{(x)\left[-\left(\frac{y}{x}\right)^{2/3}\right]-y}{x^2}\right)$$
$$=-\frac{2}{3}x^{1/3}y^{-1/3}(-x^{-5/3}y^{2/3}-x^{-2}y)$$
$$=\frac{2}{3}x^{-4/3}y^{1/3}+\frac{2}{3}x^{-5/3}y^{2/3}$$

43. $y'=2x^3-3x-1,$
$y''=6x^2-3,$
$y'''=12x,$
$y^{(4)}=12,$ and the rest are all zero.

44. $y'=\frac{x^4}{24},$
$y''=\frac{x^3}{6},$
$y'''=\frac{x^2}{2},$
$y^{(4)}=x,$
$y^{(5)}=1,$ and the rest are all zero.

45. $$\frac{dy}{dx}=\frac{d}{dx}\sqrt{x^2-2x}=\frac{1}{2\sqrt{x^2-2x}}(2x-2)=\frac{x-1}{\sqrt{x^2-2x}}$$
At $x=3$, we have $y=\sqrt{3^2-2(3)}=\sqrt{3}$
and $\frac{dy}{dx}=\frac{3-1}{\sqrt{3^2-2(3)}}=\frac{2}{\sqrt{3}}.$

(a) Tangent: $y=\frac{2}{\sqrt{3}}(x-3)+\sqrt{3}$ or $y=\frac{2}{\sqrt{3}}x-\sqrt{3}$

(b) Normal: $y=-\frac{\sqrt{3}}{2}(x-3)+\sqrt{3}$ or $y=-\frac{\sqrt{3}}{2}x+\frac{5\sqrt{3}}{2}$

46. $$\frac{dy}{dx}=\frac{d}{dx}(4+\cot x-2\csc x)$$
$$=-\csc^2 x+2\csc x\cot x$$
At $x=\frac{\pi}{2}$, we have
$y=4+\cot\frac{\pi}{2}-2\csc\frac{\pi}{2}=4+0-2=2$ and
$\frac{dy}{dx}=-\csc^2\frac{\pi}{2}+2\csc\frac{\pi}{2}\cot\frac{\pi}{2}=-1+2(1)(0)=-1.$

(a) Tangent: $y=-1\left(x-\frac{\pi}{2}\right)+2$ or $y=-x+\frac{\pi}{2}+2$

(b) Normal: $y=1\left(x-\frac{\pi}{2}\right)+2$ or $y=x-\frac{\pi}{2}+2$

47. Use implicit differentiation.
$$x^2+2y^2=9$$
$$\frac{d}{dx}(x^2)+\frac{d}{dx}(2y^2)=\frac{d}{dx}(9)$$
$$2x+4y\frac{dy}{dx}=0$$
$$\frac{dy}{dx}=-\frac{2x}{4y}=-\frac{x}{2y}.$$
Slope at (1, 2): $-\frac{1}{2(2)}=-\frac{1}{4}$

(a) Tangent: $y=-\frac{1}{4}(x-1)+2$ or $y=-\frac{1}{4}x+\frac{9}{4}$

(b) Normal: $y=4(x-1)+2$ or $y=4x-2$

48. Use implicit differentiation.
$$x+\sqrt{xy}=6$$
$$\frac{d}{dx}(x)+\frac{d}{dx}(\sqrt{xy})=\frac{d}{dx}(6)$$
$$1+\frac{1}{2\sqrt{xy}}\left[(x)\left(\frac{dy}{dx}\right)+(y)(1)\right]=0$$
$$\frac{x}{2\sqrt{xy}}\frac{dy}{dx}=-1-\frac{y}{2\sqrt{xy}}$$
$$\frac{dy}{dx}=\frac{2\sqrt{xy}}{x}\left(-1-\frac{y}{2\sqrt{xy}}\right)$$
$$=-2\sqrt{\frac{y}{x}}-\frac{y}{x}$$
Slope at (4, 1): $-2\sqrt{\frac{1}{4}}-\frac{1}{4}=-\frac{2}{2}-\frac{1}{4}=-\frac{5}{4}$

(a) Tangent: $y=-\frac{5}{4}(x-4)+1$ or $y=-\frac{5}{4}x+6$

(b) Normal: $y=\frac{4}{5}(x-4)+1$ or $y=\frac{4}{5}x-\frac{11}{5}$

49. $\dfrac{dy}{dx}=\dfrac{\frac{dy}{dt}}{\frac{dx}{dt}}=\dfrac{-2\sin t}{2\cos t}=-\tan t$

At $t=\frac{3\pi}{4}$, we have $x=2\sin\frac{3\pi}{4}=\sqrt{2}$,

$y=2\cos\frac{3\pi}{4}=-\sqrt{2}$, and $\frac{dy}{dx}=-\tan\frac{3\pi}{4}=1$.

The equation of the tangent line is

$y=1(x-\sqrt{2})+(-\sqrt{2})$, or $y=x-2\sqrt{2}$.

50. $\dfrac{dy}{dx}=\dfrac{\frac{dy}{dt}}{\frac{dx}{dt}}=\dfrac{4\cos t}{-3\sin t}=-\dfrac{4}{3}\cot t$

At $t=\frac{3\pi}{4}$, we have $x=3\cos\frac{3\pi}{4}=-\frac{3\sqrt{2}}{2}$,

$y=4\sin\frac{3\pi}{4}=2\sqrt{2}$, and $\frac{dy}{dx}=-\frac{4}{3}\cot\frac{3\pi}{4}=\frac{4}{3}$.

The equation of the tangent line is

$y=\frac{4}{3}\left(x+\frac{3\sqrt{2}}{2}\right)+2\sqrt{2}$, or $y=\frac{4}{3}x+4\sqrt{2}$.

51. $\dfrac{dy}{dx}=\dfrac{\frac{dy}{dt}}{\frac{dx}{dt}}=\dfrac{5\sec^2 t}{3\sec t\tan t}=\dfrac{5\sec t}{3\tan t}=\dfrac{5}{3\sin t}$

At $t=\frac{\pi}{6}$, we have $x=3\sec\frac{\pi}{6}=2\sqrt{3}$,

$y=5\tan\frac{\pi}{6}=\frac{5\sqrt{3}}{3}$, and $\frac{dy}{dx}=\dfrac{5}{3\sin\left(\frac{\pi}{6}\right)}=\frac{10}{3}$.

The equation of the tangent line is

$y=\frac{10}{3}(x-2\sqrt{3})+\frac{5\sqrt{3}}{3}$, or $y=\frac{10}{3}x-5\sqrt{3}$.

52. $\dfrac{dy}{dx}=\dfrac{\frac{dy}{dt}}{\frac{dx}{dt}}=\dfrac{1+\cos t}{-\sin t}$

At $t=-\frac{\pi}{4}$, we have $x=\cos\left(-\frac{\pi}{4}\right)=\frac{\sqrt{2}}{2}$,

$y=-\frac{\pi}{4}+\sin\left(-\frac{\pi}{4}\right)=-\frac{\pi}{4}-\frac{\sqrt{2}}{2}$, and

$\dfrac{dy}{dx}=\dfrac{1+\cos\left(-\frac{\pi}{4}\right)}{-\sin\left(-\frac{\pi}{4}\right)}=\dfrac{1+\frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}}=\sqrt{2}+1$.

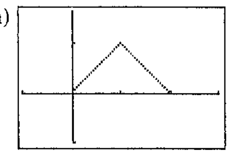
The equation of the tangent line is

$y=(\sqrt{2}+1)\left(x-\frac{\sqrt{2}}{2}\right)-\frac{\pi}{4}-\frac{\sqrt{2}}{2}$, or

$y=(1+\sqrt{2})x-\sqrt{2}-1-\frac{\pi}{4}$.

This is approximately $y=2.414x-3.200$.

53. **(a)**



[-1, 3] by [-1, 5/3]

(b) Yes, because both of the one-sided limits as $x\to 1$ are equal to $f(1)=1$.

(c) No, because the left-hand derivative at $x=1$ is $+1$ and the right-hand derivative at $x=1$ is -1.

54. **(a)** The function is continuous for all values of m, because the right-hand limit as $x\to 0$ is equal to $f(0)=0$ for any value of m.

(b) The left-hand derivative at $x=0$ is $2\cos(2\bullet 0)=2$, and the right-hand derivative at $x=0$ is m, so in order for the function to be differentiable at $x=0$, m must be 2.

55. **(a)** For all $x\neq 0$ **(b)** At $x=0$

(c) Nowhere

56. **(a)** For all x **(b)** Nowhere

(c) Nowhere

57. Note that $\lim_{x\to 0^-}f(x)=\lim_{x\to 0^-}(2x-3)=-3$ and $\lim_{x\to 0^+}f(x)=\lim_{x\to 0^+}(x-3)=-3$. Since these values agree with $f(0)$, the function is continuous at $x=0$. On the other hand,

$f'(x)=\begin{cases}2, & -1\le x<0\\ 1, & 0<x\le 4\end{cases}$, so the derivative is undefined at $x=0$.

(a) $[-1,0)\cup(0,4]$ **(b)** At $x=0$

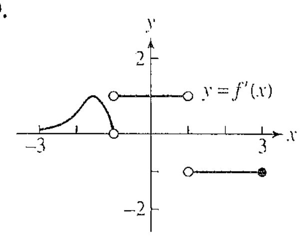
(c) Nowhere in its domain

58. Note that the function is undefined at $x=0$.

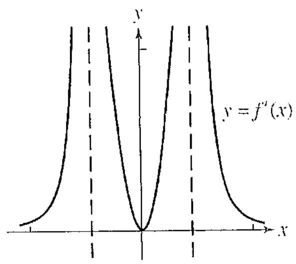
(a) $[-2,0)\cup(0,2]$ **(b)** Nowhere

(c) Nowhere in its domain

59.



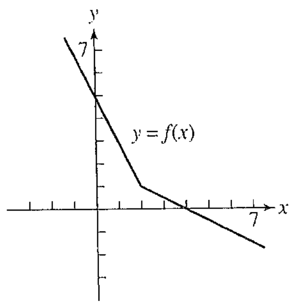
60.



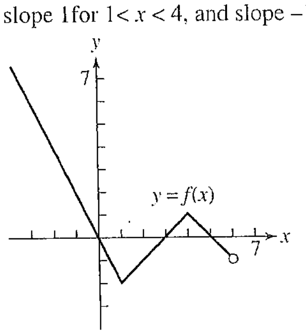
61. (a) iii **(b)** i

(c) ii

62. The graph passes through (0, 5) and has slope -2 for $x < 2$ and slope -0.5 for $x > 2$.



63. The graph passes through $(-1, 2)$ and has slope -2 for $x < 1$, slope 1 for $1 < x < 4$, and slope -1 for $4 < x < 6$.



64. i. If $f(x) = \frac{9}{28}x^{7/3} + 9$, then $f'(x) = \frac{3}{4}x^{4/3}$ and $f''(x) = x^{1/3}$, which matches the given equation.

ii. If $f'(x) = \frac{9}{28}x^{7/3} - 2$, then $f''(x) = \frac{3}{4}x^{4/3}$, which contradicts the given equation.

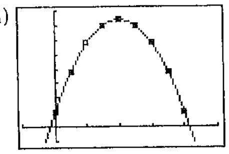
iii. If $f'(x) = \frac{3}{4}x^{4/3} + 6$, then $f''(x) = x^{1/3}$, which matches the given equation.

iv. If $f(x) = \frac{3}{4}x^{4/3} - 4$, then $f'(x) = x^{1/3}$ and

$f''(x) = \frac{1}{3}x^{-2/3}$, which contradicts the given equation.

Answer is D: i and iii only could be true. Note, however that i and iii could not simultaneously be true.

65. (a)

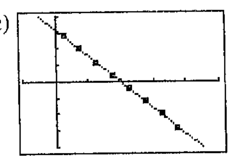


[-1, 5] by [-10, 80]

(b)

t interval	avg. vel.
[0, 0.5]	$\frac{38-10}{0.5-0} = 56$
[0.5, 1]	$\frac{58-38}{1-0.5} = 40$
[1, 1.5]	$\frac{70-58}{1.5-1} = 24$
[1.5, 2]	$\frac{74-70}{2-1.5} = 8$
[2, 2.5]	$\frac{70-74}{2.5-2} = -8$
[2.5, 3]	$\frac{58-70}{3-2.5} = -24$
[3, 3.5]	$\frac{38-58}{3.5-3} = -40$
[3.5, 4]	$\frac{10-38}{4-3.5} = -56$

(c)



[-1, 5] by [-80, 80]

(d) Average velocity is a good approximation to velocity.

66. (a) $\frac{d}{dx}\left[\sqrt{x}f(x)\right] = \sqrt{x}f'(x) + \frac{1}{2\sqrt{x}}f(x)$

At $x = 1$, the derivative is

$\sqrt{1}f'(1) + \frac{1}{2\sqrt{1}}f(1) = 1\left(\frac{1}{5}\right) + \left(\frac{1}{2}\right)(-3) = -\frac{13}{10}.$

(b) $\frac{d}{dx}\sqrt{f(x)} = \frac{1}{2\sqrt{f(x)}}f'(x) = \frac{f'(x)}{2\sqrt{f(x)}}$

At $x = 0$, the derivative is $\frac{f'(0)}{2\sqrt{f(0)}} = -\frac{2}{2\sqrt{9}} = -\frac{1}{3}.$

(c) $\frac{d}{dx}f(\sqrt{x}) = f'(\sqrt{x})\frac{d}{dx}\sqrt{x} = \frac{f'(\sqrt{x})}{2\sqrt{x}}$

At $x = 1$, the derivative is $\frac{f'(\sqrt{1})}{2\sqrt{1}} = \frac{f'(1)}{2} = \frac{\frac{1}{5}}{2} = \frac{1}{10}.$

(d) $\frac{d}{dx}f(1 - 5\tan x) = f'(1 - 5\tan x)(-5\sec^2 x)$

At $x = 0$, the derivative is

$f'(1 - 5\tan 0)(-5\sec^2 0) = f'(1)(-5) = \left(\frac{1}{5}\right)(-5) = -1.$

66. Continued

(e) $\frac{d}{dx}\frac{f(x)}{2+\cos x} = \frac{(2+\cos x)(f'(x))-(f(x))(-\sin x)}{(2+\cos x)^2}$

At $x = 0$, the derivative is

$\frac{(2+\cos 0)(f'(0))-(f(0))(-\sin 0)}{(2+\cos 0)^2} = \frac{3f'(0)}{3^2} = -\frac{2}{3}$.

(f) $\frac{d}{dx}\left[10\sin\left(\frac{\pi x}{2}\right)f^2(x)\right]$

$= 10\left(\sin\frac{\pi x}{2}\right)(2f(x)f'(x)) + 10f^2(x)\left(\cos\frac{\pi x}{2}\right)\left(\frac{\pi}{2}\right)$

$= 20f(x)f'(x)\sin\frac{\pi x}{2} + 5\pi f^2(x)\cos\frac{\pi x}{2}$

At $x = 1$, the derivative is

$20f(1)f'(1)\sin\frac{\pi}{2} + 5\pi f^2(1)\cos\frac{\pi}{2}$

$= 20(-3)\left(\frac{1}{5}\right)(1) + 5\pi(-3)^2(0)$

$= -12$.

67. (a) $\frac{d}{dx}\left[3f(x) - g(x)\right] = 3f'(x) - g'(x)$

At $x = -1$, the derivative is

$3f'(-1) - g'(-1) = 3(2) - 1 = 5$.

(b) $\frac{d}{dx}\left[f^2(x)g^3(x)\right]$

$= f^2(x) \cdot 3g^2(x)g'(x) + g^3(x) \cdot 2f(x)f'(x)$

$= f(x)g^2(x)\left[3f(x)g'(x) + 2g(x)f'(x)\right]$

At $x = 0$, the derivative is

$f(0)g^2(0)\left[3f(0)g'(0) + 2g(0)f'(0)\right]$

$= (-1)(-3)^2\left[3(-1)(4) + 2(-3)(-2)\right]$

$= -9\left[-12 + 12\right] = 0$.

(c) $\frac{d}{dx}g(f(x)) = g'(f(x))f'(x)$

At $x = -1$, the derivative is

$g'(f(-1))f'(-1) = g'(0)f'(-1) = (4)(2) = 8$.

(d) $\frac{d}{dx}f(g(x)) = f'(g(x))g'(x)$

At $x = -1$, the derivative is

$f'(g(-1))g'(-1) = f'(-1)g'(-1) = (2)(1) = 2$.

(e) $\frac{d}{dx}\frac{f(x)}{g(x)+2} = \frac{(g(x)+2)f'(x) - f(x)g'(x)}{(g(x)+2)^2}$

At $x = 0$, the derivative is

$\frac{(g(0)+2)f'(0) - f(0)g'(0)}{(g(0)+2)^2} = \frac{(-3+2)(-2)-(-1)(4)}{(-3+2)^2}$

$= 6$.

(f) $\frac{d}{dx}g(x+f(x)) = g'(x+f(x))\frac{d}{dx}(x+f(x))$

$= g'(x+f(x))(1+f'(x))$

At $x = 0$, the derivative is $g'(0+f(0))[1+f'(0)]$

$= g'(0-1)[1+(-2)] = (1)(-1) = -1$

68. $\frac{dw}{ds} = \frac{dw}{dr}\frac{dr}{ds} = \frac{d}{dr}[\sin(\sqrt{r}-2)]\frac{d}{ds}\left[8\sin\left(s+\frac{\pi}{6}\right)\right]$

$= \left[\cos(\sqrt{r}-2)\frac{1}{2\sqrt{r}}\right]\left[8\cos\left(s+\frac{\pi}{6}\right)\right]$

At $s = 0$, we have $r = 8\sin\left(0+\frac{\pi}{6}\right) = 4$ and so

$\frac{dw}{ds} = \left[\cos(\sqrt{4}-2)\frac{1}{2\sqrt{4}}\right]\left[8\cos\left(0+\frac{\pi}{6}\right)\right]$

$= \left(\frac{\cos 0}{4}\right)\left(8\cos\frac{\pi}{6}\right) = \left(\frac{1}{4}\right)\left(\frac{8\sqrt{3}}{2}\right) = \sqrt{3}$

69. Solving $\theta^2 t + \theta = 1$ for t, we have

$t = \frac{1-\theta}{\theta^2} = \theta^{-2} - \theta^{-1}$, and we may write:

$$\frac{dr}{d\theta} = \frac{dr}{dt}\frac{dt}{d\theta}$$

$$\frac{d}{d\theta}(\theta^2+7)^{1/3} = \frac{dr}{dt}\frac{d}{d\theta}(\theta^{-2}-\theta^{-1})$$

$$\frac{1}{3}(\theta^2+7)^{-2/3}(2\theta) = \left(\frac{dr}{dt}\right)(-2\theta^{-3}+\theta^{-2})$$

$$\frac{dr}{dt} = \frac{2\theta(\theta^2+7)^{-2/3}}{3(-2\theta^{-3}+\theta^{-2})} = \frac{2\theta^4(\theta^2+7)^{-2/3}}{3(\theta-2)}$$

At $t = 0$, we may solve $\theta^2 t + \theta = 1$ to obtain $\theta = 1$,

and so $\frac{dr}{dt} = \frac{2(1)^4(1^2+7)^{-2/3}}{3(1-2)} = \frac{2(8)^{-2/3}}{-3} = -\frac{1}{6}$.

70. (a) One possible answer:

$x(t) = 10\cos\left(t+\frac{\pi}{4}\right)$, $y(t) = 0$

(b) $s(0) = 10\cos\frac{\pi}{4} = 5\sqrt{2}$

(c) Farthest left:

When $\cos\left(t+\frac{\pi}{4}\right) = -1$, we have $s(t) = -10$.

Farthest right:

When $\cos\left(t+\frac{\pi}{4}\right) = 1$, we have $s(t) = 10$.

70. Continued

(d) Since $\cos\frac{\pi}{2}=0$, the particle first reaches the origin at $t=\frac{\pi}{4}$. The velocity is given by $v(t)=-10\sin\left(t+\frac{\pi}{4}\right)$, so the velocity at $t=\frac{\pi}{4}$ is $-10\sin\frac{\pi}{2}=-10$, and the speed at $t=\frac{\pi}{4}$ is $|-10|=10$. The acceleration is given by $a(t)=-10\cos\left(t+\frac{\pi}{4}\right)$, so the acceleration at $t=\frac{\pi}{4}$ is $-10\cos\frac{\pi}{2}=0$.

71. (a) $\frac{ds}{dt}=\frac{d}{dt}(64t-16t^2)=64-32t$

$\frac{d^2s}{dt^2}=\frac{d}{dt}(64-32t)=-32$

(b) The maximum height is reached when $\frac{ds}{dt}=0$, which occurs at $t=2$ sec.

(c) When $t=0$, $\frac{ds}{dt}=64$, so the velocity is 64 ft/sec.

(d) Since $\frac{ds}{dt}=\frac{d}{dt}(64t-2.6t^2)=64-5.2t$, the maximum height is reached at $t=\frac{64}{5.2}\approx 12.3\text{sec}$. The maximum height is $s\left(\frac{64}{5.2}\right)\approx 393.8$ ft.

72. (a) Solving $160=490t^2$, it takes $\frac{4}{7}$ sec. The average velocity is $\frac{160}{4/7}=280\text{cm/sec}$.

(b) Since $v(t)=\frac{ds}{dt}=980t$, the velocity is $(980)\left(\frac{4}{7}\right)=560$ cm/sec. Since $a(t)=\frac{dv}{dt}=980$, the acceleration is 980 cm/sec^2.

73. $\frac{dV}{dx}=\frac{d}{dx}\left[\pi\left(10-\frac{x}{3}\right)x^2\right]=\frac{d}{dx}\left[\pi\left(10x^2-\frac{1}{3}x^3\right)\right]$

$=\pi(20x-x^2)$

74. (a) $r(x)=\left(3-\frac{x}{40}\right)^2 x=9x-\frac{3}{20}x^2+\frac{1}{1600}x^3$

(b) The marginal revenue is

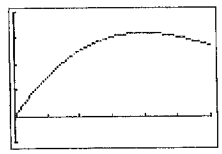
$r'(x)=9-\frac{3}{10}x+\frac{3}{1600}x^2$

$=\frac{3}{1600}(x^2-160x+4800)$

$=\frac{3}{1600}(x-40)(x-120)$,

which is zero when $x=40$ or $x=120$. Since the bus holds only 60 people, we require $0\le x\le 60$. The marginal revenue is 0 when there are 40 people, and the corresponding fare is $p(40)=\left(3-\frac{40}{40}\right)^2=\$4.00$.

(c) One possible answer:
If the current ridership is less than 40, then the proposed plan may be good. If the current ridership is greater than or equal to 40, then the plan is not a good idea. Look at the graph of $y=r(x)$.



[0, 60] by [−50, 200]

75. (a) Since $x=\tan\theta$, we have $\frac{dx}{dt}=(\sec^2\theta)\frac{d\theta}{dt}=-0.6\sec^2\theta$. At point A, we have $\theta=0$ and $\frac{dx}{dt}=-0.6\sec^2 0=-0.6$ km/sec.

(b) $0.6\ \frac{\text{rad}}{\text{sec}}\cdot\frac{1\text{ revolution}}{2\pi\text{ rad}}\cdot\frac{60\text{sec}}{1\text{min}}=\frac{18}{\pi}$ revolutions per minute or approximately 5.73 revolutions per minute.

76. Let $f(x)=\sin(x-\sin x)$. Then

$f'(x)=\cos(x-\sin x)\frac{d}{dx}(x-\sin x)$

$=\cos(x-\sin x)(1-\cos x)$. This derivative is zero when $\cos(x-\sin x)=0$ (which we need not solve) or when $\cos x=1$, which occurs at $x=2k\pi$ for integers k. For each of these values, $f(x)=f(2k\pi)=\sin(2k\pi-\sin 2k\pi)=\sin(2k\pi-0)=0$. Thus, $f(x)=f'(x)=0$ for $x=2k\pi$, which means that the graph has a horizontal tangent at each of these values of x.

77. $y'(r)=\frac{d}{dr}\left(\frac{1}{2rl}\sqrt{\frac{T}{\pi d}}\right)=\left(\frac{1}{2l}\sqrt{\frac{T}{\pi d}}\right)\frac{d}{dr}\left(\frac{1}{r}\right)=-\frac{1}{2r^2 l}\sqrt{\frac{T}{\pi d}}$

$y'(l)=\frac{d}{dl}\left(\frac{1}{2rl}\sqrt{\frac{T}{\pi d}}\right)=\left(\frac{1}{2r}\sqrt{\frac{T}{\pi d}}\right)\frac{d}{dl}\left(\frac{1}{l}\right)=-\frac{1}{2rl^2}\sqrt{\frac{T}{\pi d}}$

$y'(d)=\frac{d}{dd}\left(\frac{1}{2rl}\sqrt{\frac{T}{\pi d}}\right)=\left(\frac{1}{2rl}\sqrt{\frac{T}{\pi}}\right)\frac{d}{dd}(d^{-1/2})$

$=\frac{1}{2rl}\sqrt{\frac{T}{\pi}}\left(-\frac{1}{2}d^{-3/2}\right)=-\frac{1}{4rl}\sqrt{\frac{T}{\pi d^3}}$

$y'(T)=\frac{d}{dT}\left(\frac{1}{2rl}\sqrt{\frac{T}{\pi d}}\right)=\left(\frac{1}{2rl}\sqrt{\frac{1}{\pi d}}\right)\frac{d}{dT}(\sqrt{T})$

$=\frac{1}{2rl}\sqrt{\frac{1}{\pi d}}\left(\frac{1}{2\sqrt{T}}\right)=\frac{1}{4rl\sqrt{\pi d T}}$

Since $y'(r)<0, y'(l)<0$, and $y'(d)<0$, increasing r, l, or d would decrease the frequency. Since $y'(T)>0$, increasing T would increase the frequency.

78. **(a)** $P(0)=\frac{200}{1+e^5}\approx 1$ student

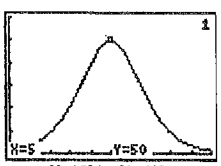
(b) $\lim_{h\to\infty}P(t)=\lim_{h\to\infty}\frac{200}{1+e^{5-t}}=\frac{200}{1}=200$ students

(c) $P'(t)=\frac{d}{dt}200(1+e^{5-t})^{-1}$

$=-200(1+e^{5-t})^{-2}(e^{5-t})(-1)$

$=\frac{200e^{5-t}}{(1+e^{5-t})^2}$

A graph of the derivative $y=P'(t)$ shows a maximum value at $t=5$, at which point $P'(t)=50$. The spread of the disease is greatest at $t=5$, when the rate is 50 students/day.

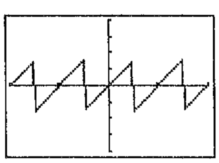


[0, 10] by [0, 60]

The maximum rate occurs at $t=5$, and this rate is

$P'(5)=\frac{200e^0}{(1+e^0)^2}=\frac{200}{2^2}=50$ students per day.

79.


[$-\pi$, π] by [-4, 4]

(a) $x\neq k\frac{\pi}{4}$, where k is an odd integer

(b) $\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$

(c) Where it's not defined, at $x=k\frac{\pi}{4}$, k an odd integer

(d) It has period $\frac{\pi}{2}$ and continues to repeat the pattern seen in this window.

80. Use implicit differentiation.

$x^2-y^2=1$

$\frac{d}{dx}(x^2)-\frac{d}{dx}(y^2)=\frac{d}{dx}(1)$

$2x-2yy'=0$

$y'=\frac{2x}{2y}=\frac{x}{y}$

$y''=\frac{d}{dx}\frac{x}{y}$

$=\frac{(y)(1)-(x)(y')}{y^2}$

$=\frac{y-x\left(\frac{x}{y}\right)}{y^2}$

$=\frac{y^2-x^2}{y^3}$

$=-\frac{1}{y^3}$

(since the given equation is $x^2-y^2=1$)

At $(2,\sqrt{3})$, $\frac{d^2y}{dx^2}=-\frac{1}{y^3}=-\frac{1}{(\sqrt{3})^3}=-\frac{1}{3\sqrt{3}}$.

81. **(a)** $v(t)=s'(t)=3t^2-12$

(b) $a(t)=v'(t)=6t$

(c) Set $v(t)=0$ and solve for t:

$3t^2-12=0$

$3(t^2-4)=0$

$3(t-2)(t+2)=0$

$t=2$ or ~~$t=-2$~~

The particle is at rest when $t=2$.

(d) $a(t)=0$ when $t=0$

speed $=|v(0)|=|3(0)^2-12|=12$

(e) Towards the origin:

$s(3)=3^3-12(3)+5=-4<0$

$v(3)=3(3)^2-12=15>0$

The particle is left of the origin and it is moving to the right.

82. (a) $\frac{dy}{dx} = \frac{d}{dx}\left(\frac{1}{2}(e^x + e^{-x})\right) = \frac{1}{2}(e^x + e^{-x}(-1)) = \frac{e^x - e^{-x}}{2}$

(b) $\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{1}{2}(e^x - e^{-x})\right) = \frac{1}{2}(e^x - e^{-x}(-1)) = \frac{e^x + e^{-x}}{2}$

(c) $y(1) = \frac{e + e^{-1}}{2} \approx 1.543$; $\left.\frac{dy}{dx}\right|_{x=1} = \frac{e - e^{-1}}{2} \approx 1.175$;

$y - 1.543 = 1.175(x - 1)$
$y = 1.175x + 0.368$

(d) $m = \frac{1}{dy/dx} = \frac{2}{e - e^{-1}} \approx \frac{1}{1.175} \approx 0.851$

$y - 1.543 = 0.851(x - 1)$
$y = 0.851x + 0.692$

(e) $\frac{e^x - e^{-x}}{2} = 0$ (set dy/dx equal to 0)

$e^x - e^{-x} = 0$ (multiply both sides by 2)

$e^x = e^{-x}$

$e^{2x} = e^0$ (multiply both sides by e^x)

$\ln e^{2x} = \ln e^0$

$2x = 0$

$x = 0$

The tangent line is horizontal at $x = 0$.

83. (a) $1 - x^2 > 0 \quad \rightarrow \quad x^2 < 1$

$\rightarrow \sqrt{x^2} < \sqrt{1} \quad \rightarrow \quad |x| < 1$

$\rightarrow -1 < x < 1$

Domain of $f = (-1, 1)$

(b) $f'(x) = \frac{1}{1 - x^2}(-2x) = -\frac{2x}{1 - x^2}$

(c) Domain of $f' = \{x \mid x^2 \neq 1 \text{ and } x \in \text{Domain of } f\}$

Domain of $f' = (-1, 1)$

(d) $f''(x) = -\frac{(1 - x^2)(2) - (-2x)(2x)}{(1 - x^2)^2}$

$= -\frac{2 - 2x^2 + 4x^2}{(1 - x^2)^2}$

$= -\frac{2 + 2x^2}{(1 - x^2)^2}$

$= -\frac{2(x^2 + 1)}{(x^2 - 1)^2} < 0$ for $x \neq \pm 1$

(The numerator and denominator are clearly both positive.) Therefore, $f''(x) < 0$ for all $x \in (-1, 1)$.

Chapter 4

Applications of Derivatives

Section 4.1 Extreme Values of Functions (pp. 187–195)

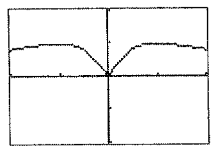
Exploration 1 Finding Extreme Values

1. From the graph we can see that there are three critical points: $x = -1, 0, 1$.

   Critical point values: $f(-1) = 0.5$, $f(0) = 0$, $f(1) = 0.5$

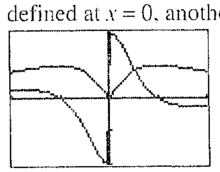
   Endpoint values: $f(-2) = 0.4$, $f(2) = 0.4$

   Thus f has absolute maximum value of 0.5 at $x = -1$ and $x = 1$, absolute minimum value of 0 at $x = 0$, and local minimum value of 0.4 at $x = -2$ and $x = 2$.

   $[-2, 2]$ by $[-1, 1]$

2. The graph of f' has zeros at $x = -1$ and $x = 1$ where the graph of f has local extreme values. The graph of f' is not defined at $x = 0$, another extreme value of the graph of f.

   $[-2, 2]$ by $[-1, 1]$

3. We can write $f(x) = \begin{cases} \frac{-x}{x^2 + 1} & \text{for } x < 0 \\ \frac{x}{x^2 + 1} & \text{for } x \geq 0 \end{cases}$,

   so the Quotient Rule gives

   $f'(x) = \begin{cases} -\frac{1 - x^2}{(x^2 + 1)^2} & \text{for } x < 0 \\ \frac{1 - x^2}{(x^2 + 1)^2} & \text{for } x \geq 0 \end{cases}$,

   which can be written as $f'(x) = \frac{|x|}{x} \cdot \frac{1 - x^2}{(x^2 + 1)^2}$.

Quick Review 4.1

1. $f'(x) = \frac{1}{2\sqrt{4 - x}} \cdot \frac{d}{dx}(4 - x) = \frac{-1}{2\sqrt{4 - x}}$

2. $f'(x) = \frac{d}{dx} 2(9 - x^2)^{-1/2} = -(9 - x^2)^{-3/2} \cdot \frac{d}{dx}(9 - x^2)$

   $= -(9 - x^2)^{-3/2}(-2x) = \frac{2x}{(9 - x^2)^{3/2}}$

3. $g'(x) = -\sin(\ln x) \cdot \frac{d}{dx} \ln x = -\frac{\sin(\ln x)}{x}$