Quick Review 6.2

1. $\int_0^2 x^4 dx = \frac{1}{5}x^5\Big]_0^2 = \frac{1}{5}(2)^5 - \frac{1}{5}(0)^5 = \frac{32}{5}$

2. $\int_1^5 \sqrt{x-1}\, dx = \int_1^5 (x-1)^{1/2} dx = \frac{2}{3}(x-1)^{3/2}\Big]_1^5$
$= \frac{2}{3}(4)^{3/2} - \frac{2}{3}(0)^{3/2}$
$= \frac{2}{3}(8) = \frac{16}{3}$

3. $\frac{dy}{dx} = 3^x$

4. $\frac{dy}{dx} = 3^x$

5. $\frac{dy}{dx} = 4(x^3 - 2x^2 + 3)^3(3x^2 - 4x)$

6. $\frac{dy}{dx} = 2 \sin (4x-5) \cos (4x-5) \cdot 4$
$= 8 \sin (4x-5) \cos (4x-5)$

7. $\frac{dy}{dx} = \frac{1}{\cos x} \cdot -\sin x = -\tan x$

8. $\frac{dy}{dx} = \frac{1}{\sin x} \cdot \cos x = \cot x$

9. $\frac{dy}{dx} = \frac{1}{\sec x + \tan x} \cdot (\sec x \tan x + \sec^2 x)$
$= \frac{\sec x \tan x + \sec^2 x}{\sec x + \tan x}$
$= \frac{\sec x(\tan x + \sec x)}{\sec x + \tan x}$
$= \sec x$

10. $\frac{dy}{dx} = \frac{1}{\csc x + \cot x}(-\csc x \cot x - \csc^2 x)$
$= -\frac{\csc x \cot x + \csc^2 x}{\csc x + \cot x}$
$= -\frac{\csc x(\cot x + \csc x)}{\csc x + \cot x}$
$= -\csc x$

Section 6.2 Exercises

1. $\int(\cos x - 3x^2)\, dx = \sin x - x^3 + C$

2. $\int x^{-2} dx = -x^{-1} + C$

3. $\int\left(t^2 - \frac{1}{t^2}\right) dt = \frac{t^3}{3} + t^{-1} + C$

4. $\int \frac{dt}{t^2+1} = \tan^{-1} t + C$

5. $\int (3x^4 - 2x^{-3} + \sec^2 x)\, dx = \frac{3}{5}x^5 + x^{-2} + \tan x + C$

6. $\int (2e^x + \sec x \tan x - \sqrt{x})\, dx = 2e^x + \sec x - \frac{2}{3}x^{3/2} + C$

7. $(-\cot u + C)' = -(-\csc^2 u) = \csc^2 u$

8. $(-\csc u + C)' = -(-\csc u \cot u) = \csc u \cot u$

9. $\left(\frac{1}{2}e^{2x} + C\right)' = \frac{1}{2}e^{2x}(2) = e^{2x}$

10. $\left(\frac{1}{\ln 5}5^x + C\right)' = \frac{1}{\ln 5}\, 5^x(\ln 5) = 5^x$

11. $(\tan^{-1} u + C)' = \frac{1}{1+u^2}$

12. $(\sin^{-1} u + C)' = \frac{1}{\sqrt{1-u^2}}$

13. $\int f(u)\, du = \int \sqrt{u}\, du = \frac{2}{3}u^{3/2} + C = \frac{2}{3}x^3 + C$
$\int f(u)\, dx = \int \sqrt{u}\, dx = \int \sqrt{x^2} dx = \int x\, dx = \frac{1}{2}x^2 + C$

14. $\int f(u)\, du = \int u^2\, du = \frac{1}{3}\, u^3 + C = \frac{1}{3}x^{15} + C$
$\int f(u)\, dx = \int u^2\, dx = \int x^{10}\, dx = \frac{1}{11}x^{11} + C$

15. $\int f(u)\, du = \int e^u\, du = e^u + C = e^{7x} + C$
$\int f(u)\, du = \int e^u dx = \int e^{7x} dx = \frac{1}{7}e^{7x} + C$

16. $\int f(u)\, du = \int \sin u\, du = -\cos u + C = -\cos 4x + C$
$\int f(u)\, dx = \int \sin u\, dx = \int \sin 4x\, dx = -\frac{1}{4}\cos 4x + C$

17. $u = 3x$
$du = 3\, dx$
$\frac{1}{3}du = dx$
$\int \sin 3x\, dx = \frac{1}{3}\int \sin u\, du$
$= -\frac{1}{3}\cos u + C$
$= -\frac{1}{3}\cos 3x + C$

Check: $\frac{d}{dx}\left(-\frac{1}{3}\cos 3x + C\right) = -\frac{1}{3}(-\sin 3x)(3) = \sin 3x$

18. $u = 2x^2$

$du = 4x\,dx$

$x\,dx = \frac{1}{4}du$

$$\int x\cos(2x^2)\,dx = \frac{1}{4}\int \cos u\,du = \frac{1}{4}\sin u + C = \frac{1}{4}\sin(2x^2) + C$$

Check: $\frac{d}{dx}\left(\frac{1}{4}\sin(2x^2) + C\right) = \frac{1}{4}\cos(2x^2)(4x) = x\cos(2x^2)$

19. $u = 2x$

$du = 2\,dx$

$\frac{1}{2}du = dx$

$$\int \sec 2x \tan 2x\,dx = \frac{1}{2}\int \sec u \tan u\,du = \frac{1}{2}\sec u + C = \frac{1}{2}\sec 2x + C$$

Check: $\frac{d}{dx}\left(\frac{1}{2}\sec 2x + C\right) = \frac{1}{2}\sec 2x \tan 2x \cdot 2 = \sec 2x \tan 2x$

20. $u = 7x - 2$

$du = 7dx$

$\frac{1}{7}du = dx$

$$\int 28(7x-2)^3 dx = \frac{1}{7}\int 28u^3 du = u^4 + C = (7x-2)^4 + C$$

Check: $\frac{d}{dx}\left[(7x-2)^4 + C\right] = 4(7x-2)^3(7) = 28(7x-2)^3$

21. $u = \frac{x}{3}$ $\quad x = 3u$

$du = \frac{1}{3}dx$ $\quad x^2 = 9u^2$

$3\,du = dx$

$$\int \frac{dx}{x^2+9} = \int \frac{3du}{9u^2+9} = \frac{3}{9}\int \frac{du}{u^2+1} = \frac{1}{3}\int \frac{du}{u^2+1} = \frac{1}{3}\tan^{-1} u + C = \frac{1}{3}\tan^{-1}\left(\frac{x}{3}\right) + C$$

Check: $\frac{d}{dx}\left(\frac{1}{3}\tan^{-1}\frac{x}{3} + C\right) = \frac{1}{3}\frac{1}{1+\left(\frac{x}{3}\right)^2} \cdot \frac{1}{3} = \frac{1}{9+x^2}$

22. $u = 1 - r^3$

$du = -3r^2\,dr$

$-\frac{1}{3}du = r^2 dr$

$$\int \frac{9r^2 dr}{\sqrt{1-r^3}} = 9\left(-\frac{1}{3}\right)\int \frac{du}{\sqrt{u}} = -3\int u^{-1/2} du = -3(2)u^{1/2} + C = -6\sqrt{1-r^3} + C$$

Check: $\frac{d}{dx}\left(-6\sqrt{1-r^3} + C\right) = -6\left(\frac{1}{2\sqrt{1-r^3}}\right)(-3r^2) = \frac{9r^2}{\sqrt{1-r^3}}$

23. $u = 1 - \cos\frac{t}{2}$

$du = \frac{1}{2}\sin\frac{t}{2}\,dt$

$2\,du = \sin\frac{t}{2}\,dt$

$$\int\left(1-\cos\frac{t}{2}\right)^2 \sin\frac{t}{2}\,dt = 2\int u^2 du = \frac{2}{3}u^3 + C = \frac{2}{3}\left(1-\cos\frac{t}{2}\right)^3 + C$$

Check: $\frac{d}{dx}\left[\frac{2}{3}\left(1-\cos\frac{t}{2}\right)^3 + C\right]$

$= 2\left(1-\cos\frac{t}{2}\right)^2\left(\sin\frac{t}{2}\right)\left(\frac{1}{2}\right)$

$= \left(1-\cos\frac{t}{2}\right)^2 \sin\frac{t}{2}$

24. $u = y^4 + 4y^2 + 1$

$du = (4y^3 + 8y)\,dy$

$du = 4(y^3 + 2y)\,dy$

$\frac{1}{4}du = (y^3 + 2y)dy$

24. Continued

$$\int 8(y^4+4y^2+1)^2(y^3+2y)dy = 8\left(\frac{1}{4}\right)\int u^2 du$$
$$= \frac{2}{3}u^3 + C$$
$$= \frac{2}{3}(y^4+4y^2+1)^3 + C$$

Check: $\frac{d}{dx}\left[\frac{2}{3}(y^4+4y^2+1)^3+C\right]$
$$= 2(y^4+4y^2+1)^2(4y^3+8y)$$
$$= 8(y^4+4y^2+1)^2(y^3+2y)$$

25. Let $u = 1-x$

$du = -dx$

$$\int \frac{dx}{(1-x)^2} = -\int \frac{du}{u^2}$$
$$= u^{-1} + C$$
$$= \frac{1}{1-x} + C$$

26. Let $u = x+2$

$du = dx$

$$\int \sec^2(x+2)\ dx = \int \sec^2 u\ du$$
$$= \tan u + C$$
$$= \tan(x+2) + C$$

27. Let $u = \tan x$

$du = \sec^2 x\ dx$

$$\int \sqrt{\tan x}\sec^2 x\ dx = \int u^{1/2} du$$
$$= \frac{2}{3}u^{3/2} + C$$
$$= \frac{2}{3}(\tan x)^{3/2} + C$$

28. Let $u = \theta + \frac{\pi}{2}$

$du = d\theta$

$$\int \sec\left(\theta+\frac{\pi}{2}\right)\tan\left(\theta+\frac{\pi}{2}\right)d\theta = \int \sec u \tan u\ du$$
$$= \sec u + C$$
$$= \sec\left(\theta+\frac{\pi}{2}\right) + C$$

29. $\int \tan(4x+2)\ dx$

$u = 4x+2$

$du = 4\ dx$

$\frac{1}{4}du = dx$

$\frac{1}{4}\int \tan u\ du$

$= -\frac{1}{4}\ln|\cos(4x+2)| + C$ or

$\frac{1}{4}\ln|\sec(4x+2)| + C$

30. $\int 3(\sin x)^{-2} dx$
$$= 3\int \frac{1}{\sin^2 x}dx$$
$$= 3\int \csc^2 x\ dx$$
$$= -3\cot x + C$$

31. Let $u = 3z+4$

$du = 3\ dz$

$\frac{1}{3}du = dz$

$$\int \cos(3z+4)dz = \frac{1}{3}\int \cos u\ du$$
$$= \frac{1}{3}\sin u + C$$
$$= \frac{1}{3}\sin(3z+4) + C$$

32. Let $u = \cot x$

$du = -\csc^2 x\ dx$

$$\int \sqrt{\cot x}\csc^2 x\ dx = -\int u^{1/2} du$$
$$= -\frac{2}{3}u^{3/2} + C$$
$$= -\frac{2}{3}(\cot x)^{3/2} + C$$

33. Let $u = \ln x$

$du = \frac{1}{x}dx$

$$\int \frac{\ln^6 x}{x}dx = \int u^6 du$$
$$= \frac{1}{7}u^7 + C$$
$$= \frac{1}{7}(\ln^7 x) + C$$

34. Let $u = \tan\left(\frac{x}{2}\right)$

$du = \frac{1}{2}\sec^2\left(\frac{x}{2}\right)dx$

$$\int \tan^7\left(\frac{x}{2}\right)\sec^2\left(\frac{x}{2}\right)dx = 2\int u^7 du$$
$$= 2 \cdot \frac{1}{8}u^8 + C$$
$$= \frac{1}{4}\tan^8\left(\frac{x}{2}\right) + C$$

35. Let $u = s^{4/3} - 8$

$$du = \frac{4}{3}s^{1/3}ds$$

$$\frac{3}{4}du = s^{1/3}ds$$

$$\int s^{1/3}\cos(s^{4/3} - 8)\,ds = \frac{3}{4}\int \cos u\,du$$
$$= \frac{3}{4}\sin u + C$$
$$= \frac{3}{4}\sin(s^{4/3} - 8) + C$$

36. $\int \frac{dx}{\sin^2 3x} = \int \csc^2 3x\,dx$

Let $u = 3x$

$du = 3\,dx$

$\frac{1}{3}du = dx$

$$\int \csc^2 3x\,dx = \frac{1}{3}\int \csc^2 u\,du$$
$$= -\frac{1}{3}\cot u + C$$
$$= -\frac{1}{3}\cot(3x) + C$$

37. Let $u = \cos(2t+1)$

$du = -\sin(2t+1)(2)dt$

$-\frac{1}{2}du = \sin(2t+1)dt$

$$\int \frac{\sin(2t+1)}{\cos^2(2t+1)}dt = -\frac{1}{2}\int u^{-2}du = \frac{1}{2}u^{-1} + C$$
$$= \frac{1}{2\cos(2t+1)} + C = \frac{1}{2}\sec(2t+1) + C$$

38. Let $u = 2 + \sin t$

$du = \cos t\,dt$

$$\int \frac{6\cos t}{(2+\sin t)^2}dt = 6\int u^{-2}du$$
$$= -6u^{-1} + C$$
$$= -\frac{6}{2+\sin t} + C$$

39. $\int \frac{dx}{x \ln x}$

$u = \ln x$

$du = \frac{dx}{x}$

$x\,du = dx$

$$\int \frac{du}{u} = \ln u = \ln(\ln x) + C$$

40. $\int \tan^2 x \sec^2 x\,dx$

$u = \tan x$

$du = \sec^2 x\,dx$

$$\int u^2 du = \frac{1}{3}u^3 + C$$

$$\frac{1}{3}\tan^3 x + C$$

41. $\int \frac{x\,dx}{x^2+1}$

$u = x^2 + 1$

$du = 2x\,dx$

$\frac{1}{2}du = x\,dx$

$$\frac{1}{2}\int \frac{du}{x^2+1} = \frac{1}{2}\ln u + C$$
$$= \frac{1}{2}\ln(x^2+1) + C$$

42. Let $u = \frac{x}{5}$ $\quad 5u = x$

$du = \frac{1}{5}dx \quad 25u^2 = x^2$

$5du = dx$

$$\int \frac{40\,dx}{x^2+25} = \int \frac{200\,du}{25u^2+5} = \frac{200}{25}\int \frac{du}{u^2+1}$$
$$= 8\tan^{-1}u + C = 8\tan^{-1}\left(\frac{x}{5}\right) + C$$

43. $\int \frac{dx}{\cot 3x} = \int \frac{\sin 3x}{\cos 3x}dx$

Let $u = \cos 3x$

$du = -3\sin 3x\,dx$

$-\frac{1}{3}du = \sin 3x\,dx$

$$\int \frac{dx}{\cot 3x} = -\frac{1}{3}\int \frac{1}{u}du$$
$$= -\frac{1}{3}\ln|u| + C$$
$$= -\frac{1}{3}\ln|\cos 3x| + C$$

(An equivalent expression is $\frac{1}{3}\ln|\sec 3x| + C$.)