

$$50. \text{NINT}\left(\frac{2x^4-1}{x^4-1}, x, -0.8, 0.8\right) \approx 1.427$$

$$51. \frac{1}{2} \text{NINT}\left(\sqrt{\cos x}, x, -1, 1\right) \approx 0.914$$

$$52. \sqrt{8-2x^2} \geq 0 \text{ between } x = -2 \text{ and } x = 2$$

$$\text{NINT}(\sqrt{8-2x^2}, x, -2, 2) \approx 8.886$$

$$53. \text{Plot } y_1 = \text{NINT}(e^{-t^2}, t, 0, x), y_2 = 0.6 \text{ in a } [0, 1] \text{ by } [0, 1] \text{ window, then use the intersect function to find } x \approx 0.699.$$

$$54. \text{When } y = 0, x = 1.$$

$$y^3 = 1 - x^3$$

$$y = \sqrt[3]{1-x^3}$$

$$\text{NINT}(\sqrt[3]{1-x^3}, x, 0, 1) \approx 0.883$$

$$55. \int_a^x f(t) dt + K = \int_b^x f(t) dt$$

$$K = -\int_a^x f(t) dt + \int_b^x f(t) dt$$

$$= \int_x^a f(t) dt + \int_b^x f(t) dt$$

$$= \int_b^a f(t) dt$$

$$K = \int_2^{-1} (t^2 - 3t + 1) dt$$

$$= \left[ \frac{1}{3}t^3 - \frac{3}{2}t^2 + t \right]_2^{-1}$$

$$= \left[ -\frac{1}{3} - \frac{3}{2} + (-1) \right] - \left[ \frac{8}{3} - 6 + 2 \right] = -\frac{3}{2}$$

56. To find an antiderivative of  $\sin^2 x$ , recall from trigonometry

$$\text{that } \cos 2x = 1 - 2 \sin^2 x, \text{ so } \sin^2 x = \frac{1}{2} - \frac{1}{2} \cos 2x.$$

$$K = \int_2^0 \sin^2 t dt$$

$$= \int_2^0 \left[ \frac{1}{2} - \frac{1}{2} \cos(2x) \right] dx$$

$$= \left[ \frac{1}{2}x - \frac{1}{4} \sin(2x) \right]_2^0$$

$$= \left[ \frac{1}{2}x - \frac{1}{2} \sin x \cos x \right]_2^0$$

$$= 0 - \left( 1 - \frac{\sin 2 \cos 2}{2} \right) = \frac{\sin 2 \cos 2 - 2}{2} \approx -1.189$$

$$57. (a) H(0) = \int_0^0 f(t) dt = 0$$

$$(b) H'(x) = \frac{d}{dx} \left( \int_0^x f(t) dt \right) = f(x)$$

$$H'(x) > 0 \text{ when } f(x) > 0.$$

$$H \text{ is increasing on } [0, 6].$$

(c)  $H$  is concave up on the open interval where

$$H''(x) = f'(x) > 0.$$

$$f'(x) > 0 \text{ when } 9 < x \leq 12.$$

$H$  is concave up on  $(9, 12)$ .

(d)  $H(12) = \int_0^{12} f(t) dt > 0$  because there is more area above the  $x$ -axis than below the  $x$ -axis.

$H(12)$  is positive.

(e)  $H'(x) = f(x) = 0$  at  $x = 6$  and  $x = 12$ . Since

$H'(x) = f(x) > 0$  on  $[0, 6)$ , the values of  $H$  are increasing to the left of  $x = 6$ , and since

$H'(x) = f(x) < 0$  on  $(6, 12]$ , the values of  $H$  are decreasing to the right of  $x = 6$ .  $H$  achieves its maximum value at  $x = 6$ .

(f)  $H(x) > 0$  on  $(0, 12]$ . Since  $H(0) = 0$ ,  $H$  achieves its minimum value at  $x = 0$ .

58. (a)  $s'(t) = f(t)$ . The velocity at  $t = 5$  is  $f(5) = 2$  units/sec.

(b)  $s''(t) = f'(t) < 0$  at  $t = 5$  since the graph is decreasing, so acceleration at  $t = 5$  is negative.

$$(c) s(3) = \int_0^3 f(x) dx = \frac{1}{2}(3)(3) = 4.5 \text{ units}$$

(d)  $s$  has its largest value at  $t = 6$  sec since  $s'(6) = f(6) = 0$  and  $s''(6) = f'(6) < 0$ .

(e) The acceleration is zero when  $s''(t) = f'(t) = 0$ . This occurs when  $t = 4$  sec and  $t = 7$  sec.

(f) Since  $s(0) = 0$  and  $s'(t) = f(t) > 0$  on  $(0, 6)$ , the particle moves away from the origin in the positive direction on  $(0, 6)$ . The particle then moves in the negative direction, towards the origin, on  $(6, 9)$  since  $s'(t) = f(t) < 0$  on  $(6, 9)$  and the area below the  $x$ -axis is smaller than the area above the  $x$ -axis.

(g) The particle is on the positive side since

$$s(9) = \int_0^9 f(x) dx > 0 \text{ (the area below the } x\text{-axis is smaller than the area above the } x\text{-axis).}$$

59. (a)  $s'(3) = f(3) = 0$  units/sec

(b)  $s''(3) = f'(3) > 0$  so acceleration is positive.

$$(c) s(3) = \int_0^3 f(x) dx = \frac{1}{2}(-6)(3) = -9 \text{ units}$$

(d)  $s(6) = \int_0^6 f(x) dx = \frac{1}{2}(-6)(3) + \frac{1}{2}(6)(3) = 0$ , so the particle passes through the origin at  $t = 6$  sec.

(e)  $s''(t) = f'(t) = 0$  at  $t = 7$  sec

(f) The particle is moving away from the origin in the negative direction on  $(0, 3)$  since  $s(0) = 0$  and  $s'(t) < 0$  on  $(0, 3)$ . The particle is moving toward the origin on  $(3, 6)$  since  $s'(t) > 0$  on  $(3, 6)$  and  $s(6) = 0$ . The particle moves away from the origin in the positive direction for  $t > 6$  since  $s'(t) > 0$ .

## 59. Continued

(g) The particle is on the positive side since

$$s(9) = \int_0^9 f(x) dx > 0$$

(the area below the  $x$ -axis is smaller than the area above the  $x$ -axis).

$$60. f(x) = \frac{d}{dx} \left( \int_1^x f(t) dt \right) = \frac{d}{dx} (x^2 - 2x + 1) = 2x - 2$$

$$61. f'(x) = \frac{d}{dx} \left( 2 + \int_0^x \frac{10}{1+t} dt \right) = \frac{10}{1+x}$$

$$f'(0) = 10$$

$$f(0) = 2 + \int_0^0 \frac{10}{1+t} dt = 2$$

$$L(x) = 2 + 10x$$

$$62. f(x) = \frac{d}{dx} \left( \int_0^x f(t) dt \right)$$

$$= \frac{d}{dx} (x \cos \pi x)$$

$$= x(-\pi \sin \pi x) + 1 \cdot \cos \pi x$$

$$= -\pi x \sin \pi x + \cos \pi x$$

$$f(4) = -4\pi \sin 4\pi + \cos 4\pi = 1$$

63. One arch of  $\sin kx$  is from  $x = 0$  to  $x = \frac{\pi}{k}$ .

$$\text{Area} = \int_0^{\pi/k} \sin kx dx = \left[ -\frac{1}{k} \cos kx \right]_0^{\pi/k} = \frac{1}{k} - \left( -\frac{1}{k} \right) = \frac{2}{k}$$

$$64. (a) \int_{-3}^2 (6 - x - x^2) dx = \left[ 6x - \frac{1}{2}x^2 - \frac{1}{3}x^3 \right]_{-3}^2$$

$$= \frac{22}{3} - \left( -\frac{27}{2} \right)$$

$$= \frac{125}{6}$$

(b) The vertex is at  $x = \frac{-(-1)}{2(-1)} = -\frac{1}{2}$ . (Recall that the vertexof a parabola  $y = ax^2 + bx + c$  is at  $x = -\frac{b}{2a}$ .)

$$y\left(-\frac{1}{2}\right) = \frac{25}{4}, \text{ so the height is } \frac{25}{4}.$$

(c) The base is  $2 - (-3) = 5$ .

$$\frac{2}{3}(\text{base})(\text{height}) = \frac{2}{3}(5)\left(\frac{25}{4}\right) = \frac{125}{6}$$

65. True. The Fundamental Theorem of Calculus guarantees that  $F$  is differentiable on  $I$ , so it must be continuous on  $I$ .66. False. In fact,  $\int_a^b e^{x^2} dx$  is a real number, so its derivative is always 0.

67. D.

68. D. See the Fundamental Theorem of Calculus.

69. E.  $f(a) + f'(a)(x - \pi)$ 

$$f(\pi) = 0$$

$$f'(\pi) = -1$$

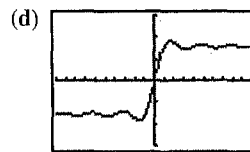
$$y = -1(x - \pi) = \pi - x$$

70. E.

71. (a)  $f(t)$  is an even function so  $\int_{-x}^0 \frac{\sin(t)}{t} dt = \int_0^x \frac{\sin(t)}{t} dt$ .

$$\begin{aligned} \text{Si}(-x) &= \int_0^{-x} \frac{\sin(t)}{t} dt \\ &= -\int_{-x}^0 \frac{\sin(t)}{t} dt \\ &= -\int_0^x \frac{\sin(t)}{t} dt = -\text{Si}(x) \end{aligned}$$

(b)  $\text{Si}(0) = \int_0^0 \frac{\sin t}{t} dt = 0$

(c)  $\text{Si}'(x) = f(x) = 0$  when  $t = \pi k$ ,  $k$  a nonzero integer.

[-20, 20] by [-3, 20, 203]

$$72. (a) c(100) - c(1) = \int_1^{100} \left( \frac{dc}{dx} \right) dx$$

$$= \int_1^{100} \frac{1}{2\sqrt{x}} dx = \left[ \sqrt{x} \right]_1^{100}$$

$$= 10 - 1 = 9 \text{ or } \$9$$

$$(b) c(400) - c(100) = \int_{100}^{400} \left( \frac{dc}{dx} \right) dx$$

$$= \int_{100}^{400} \frac{1}{2\sqrt{x}} dx = \left[ \sqrt{x} \right]_{100}^{400}$$

$$= 20 - 10 = 10 \text{ or } \$10$$

$$73. \int_0^3 \left( 2 - \frac{2}{(x+1)^2} \right) dx = \left[ 2x + 2(x+1)^{-1} \right]_0^3$$

$$= \left[ 6 + \frac{1}{2} \right] - 2 = \frac{9}{2}$$

$$= 4.5 \text{ thousand}$$

The company should expect \$4500.

$$74. (a) \frac{1}{30-0} \int_0^{30} \left( 450 - \frac{x^2}{2} \right) dx = \frac{1}{30} \left[ 450x - \frac{x^3}{6} \right]_0^{30}$$

$$= 300 \text{ drums}$$

(b)  $(300 \text{ drums})(\$0.02 \text{ per drum}) = \$6$ 75. (a) True, because  $h'(x) = f(x)$  and therefore  $h''(x) = f'(x)$ .(b) True because  $h$  and  $h'$  are both differentiable by part (a).(c) True, because  $h'(1) = f(1) = 0$ .