8. Using $A(0, -4)$, we create the parametric equations $x = 0 + at$ and $y = -4 + bt$, which determine a line passing through A at $t = 0$. We now determine a and b so that the line passes through $B(5, 0)$ at $t = 1$. Since $5 = 0 + a$, we have $a = 5$, and since $0 = -4 + b$, we have $b = 4$. Thus, one parametrization for the line segment is $x = 5t$, $y = -4 + 4t$, $0 \le t \le 1$. (Other answers are possible.)

9. One possible answer: $\frac{\pi}{2} \le t \le \frac{3\pi}{2}$

10. One possible answer: $\frac{3\pi}{2} \le t \le 2\pi$

Section 4.6 Exercises

1. Since $\frac{dA}{dt} = \frac{dA}{dr}\frac{dr}{dt}$, we have $\frac{dA}{dt} = 2\pi r\frac{dr}{dt}$,

2. Since $\frac{dS}{dt} = \frac{dS}{dr}\frac{dr}{dt}$, we have $\frac{dS}{dt} = 8\pi r\frac{dr}{dt}$.

3. **(a)** Since $\frac{dV}{dt} = \frac{dV}{dh}\frac{dh}{dt}$, we have $\frac{dV}{dt} = \pi r^2\frac{dh}{dt}$.

(b) Since $\frac{dV}{dt} = \frac{dV}{dr}\frac{dr}{dt}$, we have $\frac{dV}{dt} = 2\pi rh\frac{dr}{dt}$.

(c) $\frac{dV}{dt} = \frac{d}{dt}\pi r^2 h = \pi\frac{d}{dt}(r^2 h)$

$$\frac{dV}{dt} = \pi\left(r^2\frac{dh}{dt} + h(2r)\frac{dr}{dt}\right)$$

$$\frac{dV}{dt} = \pi r^2\frac{dh}{dt} + 2\pi rh\frac{dr}{dt}$$

4. **(a)** $\frac{dP}{dt} = \frac{d}{dt}(RI^2)$

$$\frac{dP}{dt} = R\frac{d}{dt}I^2 + I^2\frac{dR}{dt}$$

$$\frac{dP}{dt} = R\left(2I\frac{dI}{dt}\right) + I^2\frac{dR}{dt}$$

$$\frac{dP}{dt} = 2RI\frac{dI}{dt} + I^2\frac{dR}{dt}$$

(b) If P is constant, we have $\frac{dP}{dt} = 0$, which means

$2RI\frac{dI}{dt} + I^2\frac{dR}{dt} = 0$, or $\frac{dR}{dt} = -\frac{2R}{I}\frac{dI}{dt} = -\frac{2P}{I^3}\frac{dI}{dt}$.

5. $\frac{ds}{dt} = \frac{d}{dt}\sqrt{x^2 + y^2 + z^2}$

$$\frac{ds}{dt} = \frac{1}{2\sqrt{x^2 + y^2 + z^2}}\frac{d}{dt}(x^2 + y^2 + z^2)$$

$$\frac{ds}{dt} = \frac{1}{2\sqrt{x^2 + y^2 + z^2}}\left(2x\frac{dx}{dt} + 2y\frac{dy}{dt} + 2z\frac{dz}{dt}\right)$$

$$\frac{ds}{dt} = \frac{x\frac{dx}{dt} + y\frac{dy}{dt} + z\frac{dz}{dt}}{\sqrt{x^2 + y^2 + z^2}}$$

6. $\frac{dA}{dt} = \frac{d}{dt}\left(\frac{1}{2}ab\sin\theta\right)$

$$\frac{dA}{dt} = \frac{1}{2}\left(\frac{da}{dt} \cdot b \cdot \sin\theta + a \cdot \frac{db}{dt} \cdot \sin\theta + ab \cdot \frac{d}{dt}\sin\theta\right)$$

$$\frac{dA}{dt} = \frac{1}{2}\left(b\sin\theta\frac{da}{dt} + a\sin\theta\frac{db}{dt} + ab\cos\theta\frac{d\theta}{dt}\right)$$

7. **(a)** Since V is increasing at the rate of 1 volt/sec, $\frac{dV}{dt} = 1$ volt/sec.

(b) Since I is decreasing at the rate of $\frac{1}{3}$ amp/sec, $\frac{dI}{dt} = -\frac{1}{3}$ amp/sec.

(c) Differentiating both sides of $V = IR$, we have $\frac{dV}{dt} = I\frac{dR}{dt} + R\frac{dI}{dt}$.

(d) Note that $V = IR$ gives $12 = 2R$, so $R = 6$ ohms. Now substitute the known values into the equation in (c).

$$1 = 2\frac{dR}{dt} + 6\left(-\frac{1}{3}\right)$$

$$3 = 2\frac{dR}{dt}$$

$$\frac{dR}{dt} = \frac{3}{2}\text{ ohms/sec}$$

R is changing at the rate of $\frac{3}{2}$ ohms/sec. Since this value is positive, R is increasing.

8. Step 1:

r = radius of plate

A = area of plate

Step 2:

At the instant in question, $\frac{dr}{dt} = 0.01$ cm/sec, $r = 50$ cm.

Step 3:

We want to find $\frac{dA}{dt}$.

Step 4:

$A = \pi r^2$

Step 5:

$\frac{dA}{dt} = 2\pi r\frac{dr}{dt}$

Step 6:

$\frac{dA}{dt} = 2\pi(50)(0.01) = \pi\text{ cm}^2/\text{sec}$

At the instant in question, the area is increasing at the rate of $\pi\text{cm}^2/\text{sec}$.

9. Step 1:
l = length of rectangle
w = width of rectangle
A = area of rectangle
P = perimeter of rectangle
D = length of a diagonal of the rectangle

Step 2:
At the instant in question,
$\frac{dl}{dt} = -2$ cm/sec, $\frac{dw}{dt} = 2$ cm/sec, $l = 12$ cm, and $w = 5$ cm.

Step 3:
We want to find $\frac{dA}{dt}, \frac{dP}{dt}$, and $\frac{dD}{dt}$.

Steps 4, 5, and 6:

(a) $A = lw$

$$\frac{dA}{dt} = l\frac{dw}{dt} + w\frac{dl}{dt}$$
$$\frac{dA}{dt} = (12)(2) + (5)(-2) = 14 \text{ cm}^2/\text{sec}$$

The rate of change of the area is $14 \text{ cm}^2/\text{sec}$.

(b) $P = 2l + 2w$

$$\frac{dP}{dt} = 2\frac{dl}{dt} + 2\frac{dw}{dt}$$
$$\frac{dP}{dt} = 2(-2) + 2(2) = 0 \text{ cm/sec}$$

The rate of change of the perimeter is 0 cm/sec.

(c) $D = \sqrt{l^2 + w^2}$

$$\frac{dD}{dt} = \frac{1}{2\sqrt{l^2 + w^2}}\left(2l\frac{dl}{dt} + 2w\frac{dw}{dt}\right) = \frac{l\frac{dl}{dt} + w\frac{dw}{dt}}{\sqrt{l^2 + w^2}}$$
$$\frac{dD}{dt} = \frac{(12)(-2) + (5)(2)}{\sqrt{12^2 + 5^2}} = -\frac{14}{13} \text{ cm/sec}$$

The rate of change of the length of the diameter is $-\frac{14}{13}$ cm/sec.

(d) The area is increasing, because its derivative is positive. The perimeter is not changing, because its derivative is zero. The diagonal length is decreasing, because its derivative is negative.

10. Step 1:
x, y, z = edge lengths of the box
V = volume of the box
S = surface area of the box
s = diagonal length of the box

Step 2:
At the instant in question;
$\frac{dx}{dt} = 1$ m/sec, $\frac{dy}{dt} = -2$ m/sec, $\frac{dz}{dt} = 1$ m/sec, $x = 4$m, $y = 3$m, and $z = 2$m.

Step 3:
We want to find $\frac{dV}{dt}, \frac{dS}{dt}$, and $\frac{ds}{dt}$.

Steps 4, 5, and 6:

(a) $V = xyz$

$$\frac{dV}{dt} = xy\frac{dz}{dt} + xz\frac{dy}{dt} + yz\frac{dx}{dt}$$
$$\frac{dV}{dt} = (4)(3)(1) + (4)(2)(-2) + (3)(2)(1) = 2 \text{ m}^3/\text{sec}$$

The rate of change of the volume is $2 \text{ m}^3/\text{sec}$.

(b) $S = 2(xy + xz + yz)$

$$\frac{dS}{dt} 2\left(x\frac{dy}{dt} + y\frac{dx}{dt} + x\frac{dz}{dt} + z\frac{dx}{dt} + y\frac{dz}{dt} + z\frac{dy}{dt}\right)$$
$$\frac{dS}{dt} = 2[(4)(-2) + (3)(1) + (4)(1) + (2)(1) + (3)(1) + (2)(-2)] = 0 \text{ m}^2/\text{sec}$$

The rate of change of the surface area is $0 \text{ m}^2/\text{sec}$.

(c) $s = \sqrt{x^2 + y^2 + z^2}$

$$\frac{ds}{dt} = \frac{1}{2\sqrt{x^2 + y^2 + z^2}}\left(2x\frac{dx}{dt} + 2y\frac{dy}{dt} + 2z\frac{dz}{dy}\right)$$
$$= \frac{x\frac{dx}{dt} + y\frac{dy}{dt} + z\frac{dz}{dt}}{\sqrt{x^2 + y^2 + z^2}}$$
$$\frac{ds}{dt} = \frac{(4)(1) + (3)(-2) + (2)(1)}{\sqrt{4^2 + 3^2 + 2^2}} = \frac{0}{\sqrt{29}} = 0 \text{ m/sec}$$

The rate of change of the diagonal length is 0 m/sec.

11. Step 1:
r = radius of spherical balloon
S = surface area of spherical balloon
V = volume of spherical balloon

Step 2:
At the instant in question, $\frac{dV}{dt} = 100\pi \text{ ft}^3/\text{min}$ and $r = 5$ ft.

Step 3:
We want to find the values of $\frac{dr}{dt}$ and $\frac{dS}{dt}$.

Steps 4, 5, and 6:

(a) $V = \frac{4}{3}\pi r^3$

$$\frac{dV}{dt} = 4\pi r^2\frac{dr}{dt}$$
$$100\pi = 4\pi(5)^2\frac{dr}{dt}$$
$$\frac{dr}{dt} = 1 \text{ ft/min}$$

The radius is increasing at the rate of 1 ft/min.

11. Continued

(b) $S = 4\pi r^2$

$$\frac{dS}{dt} = 8\pi r \frac{dr}{dt}$$

$$\frac{dS}{dt} = 8\pi(5)(1)$$

$$\frac{dS}{dt} = 40\pi \text{ ft}^2 / \text{min}$$

The surface area is increasing at the rate of 40π ft^2/min.

12. Step 1:

r = radius of spherical droplet
S = surface area of spherical droplet
V = volume of spherical droplet

Step 2:

No numerical information is given.

Step 3:

We want to show that $\frac{dr}{dt}$ is constant.

Step 4:

$S = 4\pi r^2, V = \frac{4}{3}\pi r^3, \frac{dV}{dt} = kS$ for some constant k

Steps 5 and 6:

Differentiating $V = \frac{4}{3}\pi r^3$, we have $\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$.

Substituting kS for $\frac{dV}{dt}$ and S for $4\pi r^2$, we have $kS = S\frac{dr}{dt}$, or $\frac{dr}{dt} = k$.

13. Step 1:

s = (diagonal) distance from antenna to airplane
x = horizontal distance from antenna to airplane

Step 2:

At the instant in question,

$s = 10$ mi and $\frac{ds}{dt} = 300$ mph.

Step 3:

We want to find $\frac{dx}{dt}$.

Step 4:

$x^2 + 49 = s^2$ or $x = \sqrt{s^2 - 49}$

Step 5:

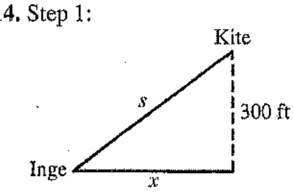
$$\frac{dx}{dt} = \frac{1}{2\sqrt{s^2-49}}\left(2s\frac{ds}{dt}\right) = \frac{s}{\sqrt{s^2-49}}\frac{ds}{dt}$$

Step 6:

$$\frac{dx}{dt} = \frac{10}{\sqrt{10^2-49}}(300) = \frac{3000}{\sqrt{51}} \text{ mph} \approx 420.08 \text{ mph}$$

The speed of the airplane is about 420.08 mph.

14. Step 1:



s = length of kite string
x = horizontal distance from Inge to kite

Step 2:

At the instant in question, $\frac{dx}{dt} = 25$ ft/sec and $s = 500$ ft

Step 3:

We want to find $\frac{ds}{dt}$.

Step 4:

$x^2 + 300^2 = s^2$

Step 5:

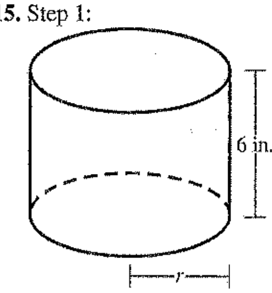
$2x\frac{dx}{dt} = 2s\frac{ds}{dt}$ or $x\frac{dx}{dt} = s\frac{ds}{dt}$

Step 6:

At the instant in question, since $x^2 + 300^2 = s^2$, we have $x = \sqrt{s^2 - 300^2} = \sqrt{500^2 - 300^2} = 400$.

Thus $(400)(25) = (500)\frac{ds}{dt}$, so $\frac{ds}{dt}$, so $\frac{ds}{dt} = 20$ ft/sec. Inge must let the string out at the rate of 20 ft/sec.

15. Step 1:



The cylinder shown represents the shape of the hole.

r = radius of cylinder
V = volume of cylinder

Step 2:

At the instant in question, $\frac{dr}{dt} = \frac{0.001 \text{ in.}}{3 \text{min}} = \frac{1}{3000}$ in./min and (since the diameter is 3.800 in.), $r = 1.900$ in.

Step 3:

We want to find $\frac{dV}{dt}$.

Step 4:

$V = \pi r^2(6) = 6\pi r^2$

Step 5:

$$\frac{dV}{dt} = 12\pi r \frac{dr}{dt}$$

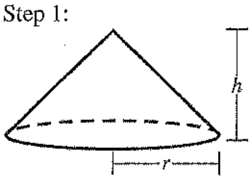
15. Continued

Step 6:

$$\frac{dV}{dt} = 12\pi(1.900)\left(\frac{1}{3000}\right) = \frac{19\pi}{2500} = 0.0076\pi$$
$$\approx 0.0239 \text{ in}^3/\text{min}.$$

The volume is increasing at the rate of approximately $0.0239 \text{ in}^3/\text{min}$.

16. Step 1:



r = base radius of cone
h = height of cone
V = volume of cone

Step 2:

At the instant in question, $h = 4$ m and $\frac{dV}{dt} = 10 \text{ m}^3/\text{min}$.

Step 3:

We want to find $\frac{dh}{dt}$ and $\frac{dr}{dt}$.

Step 4:

Since the height is $\frac{3}{8}$ of the base diameter, we have

$h = \frac{3}{8}(2r)$ or $r = \frac{4}{3}h$.

We also have $V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi\left(\frac{4}{3}h\right)^2 h = \frac{16\pi h^3}{27}$. We will use the equations $V = \frac{16\pi h^3}{27}$ and $r = \frac{4}{3}h$.

Step 5 and 6:

(a) $\frac{dV}{dt} = \frac{16\pi h^2}{9}\frac{dh}{dt}$

$$10 = \frac{16\pi(4)^2}{9}\frac{dh}{dt}$$

$$\frac{dh}{dt} = \frac{45}{128\pi}\text{m/min} = \frac{1125}{32\pi}\text{cm/min}$$

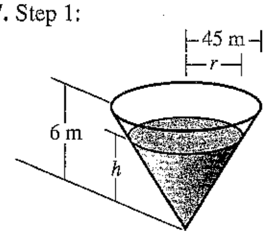
The height is changing at the rate of $\frac{1125}{32\pi} \approx 11.19$ cm/min.

(b) Using the results from Step 4 and part (a), we have

$$\frac{dr}{dt} = \frac{4}{3}\frac{dh}{dt} = \frac{4}{3}\left(\frac{1125}{32\pi}\right) = \frac{375}{8\pi}\text{cm/min}.$$

The radius is changing at the rate of $\frac{375}{8\pi} \approx 14.92$ cm/min.

17. Step 1:



r = radius of top surface of water
h = depth of water in reservoir
V = volume of water in reservoir

Step 2:

At the instant in question, $\frac{dV}{dt} = -50 \text{ m}^3/\text{min}$ and $h = 5$ m.

Step 3:

We want to find $-\frac{dh}{dt}$ and $\frac{dr}{dt}$.

Step 4:

Note that $\frac{h}{r} = \frac{6}{45}$ by similar cones, so $r = 7.5h$.

Then $V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi(7.5h)^2 h = 18.75\pi h^3$

Steps 5 and 6:

(a) Since $V = 18.75\pi h^3$, $\frac{dV}{dt} = 56.25\pi h^2\frac{dh}{dt}$.

Thus $-50 = 56.25\pi(5^2)\frac{dh}{dt}$, and

so $\frac{dh}{dt} = -\frac{8}{225\pi}$ m/min $= -\frac{32}{9\pi}$ cm/min.

The water level is falling by $\frac{32}{9\pi} \approx 1.13$ cm/min.

(Since $\frac{dh}{dt} < 0$, the rate at which the water level is *falling* is positive.)

(b) Since $r = 7.5h$, $\frac{dr}{dt} = 7.5\frac{dh}{dt} = -\frac{80}{3\pi}$ cm/min. The rate of change of the radius of the water's surface is $-\frac{80}{3\pi} \approx -8.49$ cm/min.

18. (a) Step 1:

y = depth of water in bowl
V = volume of water in bowl

Step 2:

At the instant in question, $\frac{dV}{dt} = -6 \text{ m}^3/\text{min}$ and $y = 8$ m.

Step 3:

We want to find the value of $\frac{dy}{dt}$.

Step 4:

$$V = \frac{\pi}{3}y^2(39 - y) \text{ or } V = 13\pi y^2 - \frac{\pi}{3}y^3$$

18. Continued

Step 5:

$$\frac{dV}{dt}=(26\pi y-\pi y^2)\frac{dy}{dt}$$

Step 6:

$$-6=\left[26\pi(8)-\pi(8^2)\right]\frac{dy}{dt}$$

$$-6=144\pi\frac{dy}{dt}$$

$$\frac{dy}{dt}=-\frac{1}{24\pi}\approx-0.01326\text{ m/min}$$

$$\text{or }-\frac{25}{6\pi}\approx-1.326\text{ cm/min}$$

(b) Since $r^2+(13-y)^2=13^2$,

$$r=\sqrt{169-(13-y)^2}=\sqrt{26y-y^2}.$$

(c) Step 1:

y = depth of water

r = radius of water surface

V = volume of water in bowl

Step 2:

At the instant in question, $\frac{dV}{dt}=-6\ \text{m}^3/\text{min},\ y=8$ m, and therefore (from part (a)) $\frac{dy}{dt}=-\frac{1}{24\pi}$ m/min.

Step 3:

We want to find the value of $\frac{dr}{dt}$.

Step 4:

From part (b), $r=\sqrt{26y-y^2}$.

Step 5:

$$\frac{dr}{dt}=\frac{1}{2\sqrt{26y-y^2}}(26-2y)\frac{dy}{dt}=\frac{13-y}{\sqrt{26y-y^2}}\frac{dy}{dt}$$

Step 6:

$$\frac{dr}{dt}=\frac{13-8}{\sqrt{26(8)-8^2}}\left(\frac{1}{-24\pi}\right)=\frac{5}{12}\left(-\frac{1}{24\pi}\right)$$

$$=-\frac{5}{288\pi}\approx-0.00553\text{ m/min}$$

$$\text{or }-\frac{125}{72\pi}\approx-0.553\text{ cm/min}$$

19. Step 1:

x = distance from wall to base of ladder

y = height of top of ladder

A = area of triangle formed by the ladder, wall, and ground

θ = angle between the ladder and the ground

Step 2:

At the instant in question, $x=12$ ft and $\frac{dx}{dt}=5$ ft/sec.

Step 3:

We want to find $-\frac{dy}{dt}, \frac{dA}{dt}$, and $\frac{d\theta}{dt}$.

Steps 4, 5, and 6:

(a) $x^2+y^2=169$

$$2x\frac{dx}{dt}+2y\frac{dy}{dt}=0$$

To evaluate, note that, at the instant in question,

$$y=\sqrt{169-x^2}=\sqrt{169-12^2}=5.$$

Then $2(12)(5)+2(5)\frac{dy}{dt}=0$

$$\frac{dy}{dt}=-12\text{ ft/sec}\left(\text{or }-\frac{dy}{dt}=12\text{ ft/sec}\right)$$

The top of the ladder is sliding down the wall at the rate of 12 ft/sec. (Note that the *downward* rate of motion is positive.)

(b) $A=\frac{1}{2}xy$

$$\frac{dA}{dt}=\frac{1}{2}\left(x\frac{dy}{dt}+y\frac{dx}{dt}\right)$$

Using the results from step 2 and from part (a), we have $\frac{dA}{dt}=\frac{1}{2}[(12)(-12)+(5)(5)]=-\frac{119}{2}\ \text{ft}^2/\text{sec}$. The area of the triangle is changing at the rate of $-59.5\ \text{ft}^2/\text{sec}$.

(c) $\tan\theta=\frac{y}{x}$

$$\sec^2\theta\frac{d\theta}{dt}=\frac{x\frac{dy}{dt}-y\frac{dx}{dt}}{x^2}$$

Since $\tan\theta=\frac{5}{12}$, we have

$$\left(\text{for }0\le\theta<\frac{\pi}{2}\right)\cos\theta=\frac{12}{13}\text{ and so }\sec^2\theta\frac{1}{\left(\frac{12}{13}\right)^2}=\frac{169}{144}.$$

Combining this result with the results from step 2 and from part (a), we have $\frac{169}{144}\frac{d\theta}{dt}=\frac{(12)(-12)-(5)(5)}{12^2}$, so $\frac{d\theta}{dt}=-1$ radian/sec. The angle is changing at the rate of -1 radian/sec.

20. Step 1:

h = height (or depth) of the water in the trough

V = volume of water in the trough

Step 2:

At the instant in question, $\frac{dV}{dt}=2.5\ \text{ft}^3/\text{min}$ and $h=2$ ft.

20. Continued

Step 3:

We want to find $\frac{dh}{dt}$.

Step 4:

The width of the top surface of the water is $\frac{4}{3}h$, so we have $V = \frac{1}{2}(h)\left(\frac{4}{3}h\right)(15)$, or $V = 10h^2$

Step 5:

$$\frac{dV}{dt} = 20h\frac{dh}{dt}$$

Step 6:

$$2.5 = 20(2)\frac{dh}{dt}$$

$$\frac{dh}{dt} = 0.0625 = \frac{1}{16} \text{ ft/min}$$

The water level is increasing at the rate of $\frac{1}{16}$ ft/min.

21. Step 1:

l = length of rope
x = horizontal distance from boat to dock
θ = angle between the rope and a vertical line

Step 2:

At the instant in question, $\frac{dl}{dt} = -2$ ft/sec and $l = 10$ ft.

Step 3:

We want to find the values of $-\frac{dx}{dt}$ and $\frac{d\theta}{dt}$.

Steps 4, 5, and 6:

(a) $x = \sqrt{l^2 - 36}$

$$\frac{dx}{dt} = \frac{l}{\sqrt{l^2 - 36}}\frac{dl}{dt}$$

$$\frac{dx}{dt} = \frac{10}{\sqrt{10^2 - 36}}(-2) = -2.5 \text{ ft/sec}$$

The boat is approaching the dock at the rate of 2.5 ft/sec.

(b) $\theta = \cos^{-1}\frac{6}{l}$

$$\frac{d\theta}{dt} = -\frac{1}{\sqrt{1 - \left(\frac{6}{l}\right)^2}}\left(-\frac{6}{l^2}\right)\frac{dl}{dt}$$

$$\frac{d\theta}{dt} = -\frac{1}{\sqrt{1 - 0.6^2}}\left(-\frac{6}{10^2}\right)(-2) = -\frac{3}{20} \text{ radian/sec}$$

The rate of change of angle θ is $-\frac{3}{20}$ radian/sec.

22. Step 1:

x = distance from origin to bicycle
y = height of balloon (distance from origin to balloon)
s = distance from balloon to bicycle

Step 2:

We know that $\frac{dy}{dt}$ is a constant 1 ft/sec and $\frac{dx}{dt}$ is a constant 17 ft/sec. Three seconds before the instant in question, the values of x and y are $x = 0$ ft and $y = 65$ ft. Therefore, at the instant in question $x = 51$ ft and $y = 68$ ft.

Step 3:

We want to find the value of $\frac{ds}{dt}$ at the instant in question.

Step 4:

$$s = \sqrt{x^2 + y^2}$$

Step 5:

$$\frac{ds}{dt} = \frac{1}{2\sqrt{x^2 + y^2}}\left(2x\frac{dx}{dt} + 2y\frac{dy}{dt}\right) = \frac{x\frac{dx}{dt} + y\frac{dy}{dt}}{\sqrt{x^2 + y^2}}$$

Step 6:

$$\frac{ds}{dt} = \frac{(51)(17) + (68)(1)}{\sqrt{51^2 + 68^2}} = 11 \text{ ft/sec}$$

The distance between the balloon and the bicycle is increasing at the rate of 11 ft/sec.

23. $\frac{dy}{dt} = \frac{dy}{dt}\frac{dx}{dt} = -10(1 + x^2)^{-2}(2x)\frac{dx}{dt} = -\frac{20x}{(1 + x^2)^2}\frac{dx}{dt}$

Since $\frac{dx}{dt} = 3$cm/sec, we have

$$\frac{dy}{dt} = -\frac{60x}{(1 + x^2)^2} \text{ cm/sec.}$$

(a) $\frac{dy}{dt} = -\frac{60(-2)}{[1 + (-2)^2]^2} = \frac{120}{5^2} = \frac{24}{5}$ cm/sec

(b) $\frac{dy}{dt} = -\frac{60(0)}{(1 + 0^2)^2} = 0$ cm/sec

(c) $\frac{dy}{dt} = -\frac{60(20)}{(1 + 20^2)^2} \approx -0.00746$ cm/sec

24. $\frac{dy}{dt} = \frac{dy}{dx}\frac{dx}{dt} = (3x^2 - 4)\frac{dx}{dt}$

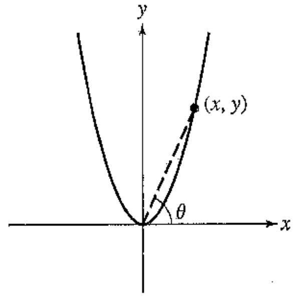
Since $\frac{dx}{dt} = -2$ cm/sec, we have $\frac{dy}{dt} = 8 - 6x^2$ cm/sec.

(a) $\frac{dy}{dt} = 8 - 6(-3)^2 = -46$ cm/sec

(b) $\frac{dy}{dt} = 8 - 6(1)^2 = 2$ cm/sec

(c) $\frac{dy}{dt} = 8 - 6(4)^2 = -88$ cm/sec

25. Step 1:



$x = x$-coordinate of particle's location
$y = y$-coordinate of particle's location
θ = angle of inclination of line joining the particle to the origin.

Step 2:
At the instant in question,
$\frac{dx}{dt} = 10$ m/sec and $x = 3$ m.

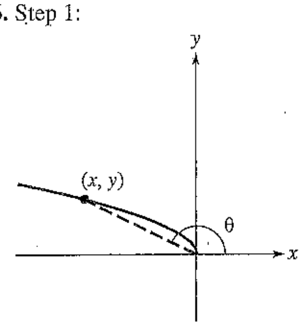
Step 3:
We want to find $\frac{d\theta}{dt}$.

Step 4:
Since $y = x^2$, we have $\tan\,\theta = \frac{y}{x} = \frac{x^2}{x} = x$ and so, for $x > 0$,
$\theta = \tan^{-1} x$.

Step 5:
$$\frac{d\theta}{dt} = \frac{1}{1+x^2}\frac{dx}{dt}$$

Step 6:
$$\frac{d\theta}{dt} = \frac{1}{1+3^2}(10) = 1 \text{ radian/sec}$$
The angle of inclination is increasing at the rate of 1 radian/sec.

26. Step 1:



$x = x$-coordinate of particle's location
$y = y$-coordinate of particle's location
θ = angle of inclination of line joining the particle to the origin

Step 2:
At the instant in question, $\frac{dx}{dt} = -8$ m / sec and $x = -4$ m.

Step 3:
We want to find $\frac{d\theta}{dt}$,

Step 4:
Since $y = \sqrt{-x}$, we have $\tan\theta = \frac{y}{x} = \frac{\sqrt{-x}}{x} - (-x)^{-1/2}$,
and so, for $x < 0$,
$\theta = \pi + \tan^{-1}[-(-x)^{1/2}] = \pi - \tan^{-1}(-x)^{-1/2}$.

Step 5:
$$\frac{d\theta}{dt} = -\frac{1}{1+[(-x)^{-1/2}]^2}\left(-\frac{1}{2}(-x)^{-3/2}(-1)\right)\frac{dx}{dt}$$
$$= -\frac{1}{1-\left(\frac{1}{x}\right)}\frac{1}{2(-x)^{3/2}}\frac{dx}{dt}$$
$$= \frac{1}{2\sqrt{-x}(x-1)}\frac{dx}{dt}$$

Step 6:
$$\frac{d\theta}{dt} = \frac{1}{2\sqrt{4}(-4-1)}(-8) = \frac{2}{5} \text{ radian/sec}$$
The angle of inclination is increasing at the rate of $\frac{2}{5}$ radian/sec.

27. Step 1:
r = radius of balls plus ice
S = surface area of ball plus ice
V = volume of ball plus ice

Step 2:
At the instant in question,
$\frac{dV}{dt} = -8$ mL/min $= -8\,\text{cm}^3$/min and $r = \frac{1}{2}(20) = 10$ cm.

Step 3:
We want to find $-\frac{dS}{dt}$.

Step 4:
We have $V = \frac{4}{3}\pi r^3$ and $S = 4\pi r^2$. These equations can be combined by noting that $r = \left(\frac{3V}{4\pi}\right)^{1/3}$, so $S = 4\pi\left(\frac{3V}{4\pi}\right)^{2/3}$

Step 5:
$$\frac{dS}{dt} = 4\pi\left(\frac{2}{3}\right)\left(\frac{3V}{4\pi}\right)^{-1/3}\left(\frac{3}{4\pi}\right)\frac{dV}{dt} = 2\left(\frac{3V}{4\pi}\right)^{-1/3}\frac{dV}{dt}$$

Step 6:
Note that $V = \frac{4}{3}\pi(10)^3 = \frac{4000\pi}{3}$.
$$\frac{dS}{dt} = 2\left(\frac{3}{4\pi}\cdot\frac{4000\pi}{3}\right)^{-1/3}(-8) = \frac{-16}{\sqrt[3]{1000}} = -1.6\,\text{cm}^2/\text{min}$$
Since $\frac{dS}{dt} < 0$, the rate of *decrease* is positive. The surface area is decreasing at the rate of $1.6\,\text{cm}^2$/min.