3. $\ln(e^{\tan x}) = \tan x$

4. $\ln(x^2-4) - \ln(x+2) = \ln\dfrac{x^2-4}{x+2}$

$= \ln\dfrac{(x+2)(x-2)}{x+2} = \ln(x-2)$

5. $\log_2(8^{x-5}) = \log_2(2^3)^{x-5} = \log_2 2^{3x-15} = 3x-15$

6. $\dfrac{\log_4 x^{15}}{\log_4 x^{12}} = \dfrac{15\log_4 x}{12\log_4 x} = \dfrac{15}{12} = \dfrac{5}{4},\ x>0$

7. $3\ln x - \ln 3x + \ln(12x^2) = \ln x^3 - \ln 3x + \ln(12x^2)$

$= \ln\dfrac{(x^3)(12x^2)}{3x} = \ln(4x^4)$

8. $3^x = 19$

$\ln 3^x = \ln 19$

$x\ln 3 = \ln 19$

$x = \dfrac{\ln 19}{\ln 3} \approx 2.68$

9. $5^t \ln 5 = 18$

$5^t = \dfrac{18}{\ln 5}$

$\ln 5^t = \ln\dfrac{18}{\ln 5}$

$t\ln 5 = \ln 18 - \ln(\ln 5)$

$t = \dfrac{\ln 18 - \ln(\ln 5)}{\ln 5} \approx 1.50$

10. $3^{x+1} = 2x$

$\ln 3^{x+1} = \ln 2^x$

$(x+1)\ln 3 = x\ln 2$

$x(\ln 3 - \ln 2) = -\ln 3$

$x = \dfrac{\ln 3}{\ln 2 - \ln 3} \approx -2.71$

Section 3.9 Exercises

1. $\dfrac{dy}{dx} = \dfrac{d}{dx}(2e^x) = 2e^x$

2. $\dfrac{dy}{dx} = \dfrac{d}{dx}(e^{2x}) = e^{2x}\dfrac{d}{dx}(2x) = 2e^{2x}$

3. $\dfrac{dy}{dx} = \dfrac{d}{dx}e^{-x} = e^{-x}\dfrac{d}{dx}(-x) = -e^{-x}$

4. $\dfrac{dy}{dx} = \dfrac{d}{dx}e^{-5x} = e^{-5x}\dfrac{d}{dx}(-5x) = -5e^{-5x}$

5. $\dfrac{dy}{dx} = \dfrac{d}{dx}e^{2x/3} = e^{2x/3}\dfrac{d}{dx}\left(\dfrac{2x}{3}\right) = \dfrac{2}{3}e^{2x/3}$

6. $\dfrac{dy}{dx} = \dfrac{d}{dx}e^{-x/4} = e^{-x/4}\dfrac{d}{dx}\left(-\dfrac{x}{4}\right) = -\dfrac{1}{4}e^{-x/4}$

7. $\dfrac{dy}{dx} = \dfrac{d}{dx}(xe^2) - \dfrac{d}{dx}(e^x) = e^2 - e^x$

8. $\dfrac{dy}{dx} = \dfrac{d}{dx}(x^2e^x) - \dfrac{d}{dx}(xe^x)$

$= (x^2)(e^x) + (e^x)(2x) - [(x)(e^x) + (e^x)(1)]$

$= x^2e^x + xe^x - e^x$

9. $\dfrac{dy}{dx} = \dfrac{d}{dx}e^{\sqrt{x}} = e^{\sqrt{x}}\dfrac{d}{dx}(\sqrt{x}) = \dfrac{e^{\sqrt{x}}}{2\sqrt{x}}$

10. $\dfrac{dy}{dx} = \dfrac{d}{dx}e^{(x^2)} = e^{(x^2)}\dfrac{d}{dx}(x^2) = 2xe^{(x^2)}$

11. $\dfrac{dy}{dx} = \dfrac{d}{dx}8^x = 8^x\ln 8$

12. $\dfrac{dy}{dx} = \dfrac{d}{dx}9^{-x} = 9^{-x}(\ln 9)\dfrac{d}{dx}(-x) = -9^{-x}\ln 9$

13. $\dfrac{dy}{dx} = \dfrac{d}{dx}3^{\csc x} = 3^{\csc x}(\ln 3)\dfrac{d}{dx}(\csc x)$

$= 3^{\csc x}(\ln 3)(-\csc x\cot x)$

$= -3^{\csc x}(\ln 3)(\csc x\cot x)$

14. $\dfrac{dy}{dx} = \dfrac{d}{dx}3^{\cot x} = 3^{\cot x}(\ln 3)\dfrac{d}{dx}(\cot x)$

$= 3^{\cot x}(\ln 3)(-\csc^2 x)$

$= -3^{\cot x}(\ln 3)(\csc^2 x)$

15. $\dfrac{dy}{dx} = \dfrac{d}{dx}\ln(x^2) = \dfrac{1}{x^2}\dfrac{d}{dx}(x^2) = \dfrac{1}{x^2}(2x) = \dfrac{2}{x}$

16. $\dfrac{dy}{dx} = \dfrac{d}{dx}(\ln x)^2 = 2\ln x\dfrac{d}{dx}(\ln x) = \dfrac{2\ln x}{x}$

17. $\dfrac{dy}{dx} = \dfrac{d}{dx}\ln(x^{-1}) = \dfrac{d}{dx}(-\ln x) = -\dfrac{1}{x},\ x>0$

18. $\dfrac{dy}{dx} = \dfrac{d}{dx}\ln\dfrac{10}{x} = \dfrac{d}{dx}(\ln 10 - \ln x) = 0 - \dfrac{1}{x}$

$= -\dfrac{1}{x},\ x>0$

19. $\dfrac{d}{dx}\ln(\ln x) = \dfrac{1}{\ln x}\dfrac{d}{dx}\ln x = \dfrac{1}{\ln x}\cdot\dfrac{1}{x} = \dfrac{1}{x\ln x}$

20. $\dfrac{dy}{dx} = \dfrac{d}{dx}(x\ln x - x) = (x)\left(\dfrac{1}{x}\right) + (\ln x)(1) - 1$

$= 1 + \ln x - 1 = \ln x$

21. $\dfrac{dy}{dx} = \dfrac{d}{dx}(\log_4 x^2) = \dfrac{d}{dx}\dfrac{\ln x^2}{\ln 4} = \dfrac{d}{dx}\left[\left(\dfrac{2}{\ln 4}\right)(\ln x)\right]$

$= \dfrac{2}{\ln 4}\cdot\dfrac{1}{x} = \dfrac{2}{x\ln 4} = \dfrac{1}{x\ln 2}$

22. $\dfrac{dy}{dx} = \dfrac{d}{dx}(\log_5\sqrt{x}) = \dfrac{d}{dx}\dfrac{\ln x^{1/2}}{\ln 5} = \dfrac{d}{dx}\dfrac{\frac{1}{2}\ln x}{\ln 5}$

$= \dfrac{1}{2\ln 5}\dfrac{d}{dx}(\ln x) = \dfrac{1}{2\ln 5}\cdot\dfrac{1}{x} = \dfrac{1}{2x\ln 5},\ x>0$

23. $\frac{dy}{dx} = \frac{d}{dx}\log_2\left(\frac{1}{x}\right) = \frac{d}{dx}(-\log_2 x) = -\frac{1}{x\ln 2}, x > 0$

24. $\frac{dy}{dx} = \frac{d}{dx}\frac{1}{\log_2 x} = -\frac{1}{(\log_2 x)^2}\frac{d}{dx}(\log_2 x)$

$= -\frac{1}{(\log_2 x)^2}\frac{1}{x\ln 2} = -\frac{1}{x(\ln 2)(\log_2 x)^2}$

or $-\frac{\ln 2}{x(\ln x)^2}$

25. $\frac{dy}{dx} = \frac{d}{dx}(\ln 2 \bullet \log_2 x) = (\ln 2)\frac{d}{dx}(\log_2 x)$

$= (\ln 2)\left(\frac{1}{x\ln 2}\right) = \frac{1}{x}, x > 0$

26. $\frac{dy}{dx} = \frac{d}{dx}\log_3(1 + x\ln 3)$

$= \frac{1}{(1 + x\ln 3)\ln 3}\frac{d}{dx}(1 + x\ln 3)$

$= \frac{\ln 3}{(1 + x\ln 3)\ln 3} = \frac{1}{1 + x\ln 3},\ x > -\frac{1}{\ln 3}$

27. $\frac{dy}{dx} = \frac{d}{dx}(\log_{10} e^x) = \frac{d}{dx}(x\log_{10} e) = \log_{10} e = \frac{\ln e}{\ln 10}$

$= \frac{1}{\ln 10}$

28. $\frac{dy}{dx} = \frac{d}{dx}\ln 10^x = \frac{d}{dx}(x\ln 10) = \ln 10$

29. $m = 5$

$y = 3^x + 1$

$y' = 3^x \ln 3 = 5$

$x = 1.379$

$y = 3^{1.379} + 1 = 5.551$

$(1.379,\ 5.551)$

30. $m_2 = -\frac{1}{m_1} = \frac{1}{3}$

$\frac{d}{dx}(2e^x - 1) = 2e^x$

$\frac{1}{3} = 2e^x$

$\frac{1}{6} = e^x$

$x = -\ln 6$

$y = 2e^x - 1$

$y = \frac{2}{6} - 1 = -\frac{2}{3}$

$\left(-\ln 6,\ -\frac{2}{3}\right)$ or

$(-1.792,\ -0.667)$

31. Equation of line: $y = mx$

Slope: $m = \frac{d}{dx}\ln(2x) = \frac{1}{2x} \bullet 2 = \frac{1}{x}$

At the point where the tangent line touches the graph, $y = mx$ and $y = \ln(2x)$

$mx = \ln(2x)$

$\frac{1}{x} \bullet x = \ln(2x)$

$1 = \ln(2x)$

$e^1 = 2x$

$x = \frac{e}{2}$

Therefore, $m = \frac{1}{x} = \frac{2}{e}$.

32. Equation of line: $y = mx$

Slope: $m = \frac{d}{dx}\left(\ln\frac{x}{3}\right) = \frac{1}{x/3}\frac{d}{dx}\left(\frac{x}{3}\right) = \frac{3}{x} \bullet \frac{1}{3} = \frac{1}{x}$

At the point where the tangent line touches the graph, $y = mx$ and $y = \ln\left(\frac{x}{3}\right)$

$mx = \ln\left(\frac{x}{3}\right)$

$\frac{1}{x} \bullet x = \ln\left(\frac{x}{3}\right)$

$1 = \ln\left(\frac{x}{3}\right)$

$e^1 = \frac{x}{3}$

$x = 3e$

Therefore, $m = \frac{1}{x} = \frac{1}{3e}$.

33. $\frac{dy}{dx} = \frac{d}{dx}(x^\pi) = \pi x^{\pi - 1}$

34. $\frac{dy}{dx} = \frac{d}{dx}(x^{1+\sqrt{2}}) = (1+\sqrt{2})x^{1+\sqrt{2}-1} = (1+\sqrt{2})x^{\sqrt{2}}$

35. $\frac{dy}{dx} = \frac{d}{dx}x^{-\sqrt{2}} = -\sqrt{2}x^{-\sqrt{2}-1}$

36. $\frac{dy}{dx} = \frac{d}{dx}x^{1-e} = (1-e)x^{1-e-1} = (1-e)x^{-e}$

37. $\frac{d}{dx}\ln(x+2) = \frac{1}{x+2}\frac{d}{dx}(x+2) = \frac{1}{x+2}$

Domain of f: $x + 2 > 0$

$x > -2$

Domain of f': $x \neq -2$ and $x > -2$, so $x > -2$