9. $f(x) = y = \dfrac{3x-2}{x}$

$xy = 3x - 2$

$(y-3)x = -2$

$x = \dfrac{-2}{y-3} = \dfrac{2}{3-y}$

Interchange x and y:

$y = \dfrac{2}{3-x}$

$f^{-1}(x) = \dfrac{2}{3-x}$

10. $f(x) = y = \arctan\dfrac{x}{3}$

$\tan y = \dfrac{x}{3},\ -\dfrac{\pi}{2} < y < \dfrac{\pi}{2}$

$x = 3\tan y,\ -\dfrac{\pi}{2} < y < \dfrac{\pi}{2}$

Interchange x and y:

$y = 3\tan x,\ -\dfrac{\pi}{2} < x < \dfrac{\pi}{2}$

$f^{-1}(x) = 3\tan x,\ -\dfrac{\pi}{2} < x < \dfrac{\pi}{2}$

Section 3.8 Exercises

1. $\dfrac{dy}{dx} = \dfrac{d}{dx}\cos^{-1}(x^2) = -\dfrac{1}{\sqrt{1-(x^2)^2}}\dfrac{d}{dx}(x^2)$

$= -\dfrac{1}{\sqrt{1-x^4}}(2x) = -\dfrac{2x}{\sqrt{1-x^4}}$

2. $\dfrac{dy}{dx} = \dfrac{d}{dx}\cos^{-1}\left(\dfrac{1}{x}\right) = -\dfrac{1}{\sqrt{1-\left(\dfrac{1}{x}\right)^2}}\dfrac{d}{dx}\left(\dfrac{1}{x}\right)$

$= -\dfrac{1}{\sqrt{1-\left(\dfrac{1}{x}\right)^2}}\left(-\dfrac{1}{x^2}\right) = \dfrac{1}{|x|\sqrt{x^2-1}}$

3. $\dfrac{dy}{dt} = \dfrac{d}{dt}\sin^{-1}\sqrt{2}t = \dfrac{1}{\sqrt{1-(\sqrt{2}t)^2}}\dfrac{d}{dt}(\sqrt{2}t) = \dfrac{\sqrt{2}}{\sqrt{1-2t^2}}$

4. $\dfrac{dy}{dt} = \dfrac{d}{dt}\sin^{-1}(1-t) = \dfrac{1}{\sqrt{1-(1-t)^2}}\dfrac{d}{dt}(1-t)$

$= -\dfrac{1}{\sqrt{2t-t^2}}$

5. $\dfrac{dy}{dt} = \dfrac{d}{dt}\sin^{-1}\left(\dfrac{3}{t^2}\right) = \dfrac{1}{\sqrt{1-\left(\dfrac{3}{t^2}\right)^2}}\dfrac{d}{dt}\left(\dfrac{3}{t^2}\right)$

$= \dfrac{1}{\sqrt{1-\dfrac{9}{t^4}}}\left(-\dfrac{6}{t^3}\right) = -\dfrac{6}{t\sqrt{t^4-9}}$

6. $\dfrac{dy}{ds} = \dfrac{d}{ds}(s\sqrt{1-s^2}) + \dfrac{d}{ds}(\cos^{-1}s)$

$= (s)\left(\dfrac{1}{2\sqrt{1-s^2}}\right)(-2s) + (\sqrt{1-s^2})(1) - \dfrac{1}{\sqrt{1-s^2}}$

$= -\dfrac{s^2}{\sqrt{1-s^2}} + \sqrt{1-s^2} - \dfrac{1}{\sqrt{1-s^2}}$

$= -\dfrac{-s^2+(1-s^2)-1}{\sqrt{1-s^2}}$

$= -\dfrac{2s^2}{\sqrt{1-s^2}}$

7. $\dfrac{dy}{dx} = \dfrac{d}{dx}(x\sin^{-1}x) + \dfrac{d}{dx}(\sqrt{1-x^2})$

$= (x)\left(\dfrac{1}{\sqrt{1-x^2}}\right) + (\sin^{-1}x)(1) + \dfrac{1}{2\sqrt{1-x^2}}(-2x)$

$= \sin^{-1}x$

8. $\dfrac{dy}{dx} = \dfrac{d}{dx}[\sin^{-1}(2x)]^{-1}$

$= -[\sin^{-1}(2x)]^{-2}\dfrac{d}{dx}\sin^{-1}(2x)$

$= -[\sin^{-1}(2x)]^{-2}\dfrac{1}{\sqrt{1-4x^2}}(2)$

$= -\dfrac{2}{[\sin^{-1}(2x)]^2\sqrt{1-4x^2}}$

9. $x(t) = \sin^{-1}\left(\dfrac{t}{4}\right)$

$\dfrac{dx}{dt} = \dfrac{d}{dt}\left[\sin^{-1}\left(\dfrac{t}{4}\right)\right] = \dfrac{1}{\sqrt{1-\left(\dfrac{t}{4}\right)^2}}\dfrac{d}{dt}\left(\dfrac{t}{4}\right)$

$= \dfrac{1}{\sqrt{1-\dfrac{t^2}{16}}} \cdot \dfrac{1}{4} = \dfrac{1}{\sqrt{16-t^2}}$

$v(3) = \left.\dfrac{dx}{dt}\right|_{t=3} = \dfrac{1}{\sqrt{16-3^2}} = \dfrac{\sqrt{7}}{7}$

10. $\dfrac{dx}{dt} = \dfrac{d}{dt}\left[\sin^{-1}\left(\dfrac{\sqrt{t}}{4}\right)\right] = \dfrac{1}{\sqrt{1-\left(\dfrac{\sqrt{t}}{4}\right)^2}}\dfrac{d}{dt}\left(\dfrac{\sqrt{t}}{4}\right)$

$= \dfrac{1}{\sqrt{1-\dfrac{t}{16}}} \cdot \dfrac{1}{8\sqrt{t}}$

$v(4) = \left.\dfrac{dx}{dt}\right|_{t=4} = \dfrac{1}{\sqrt{1-\dfrac{4}{16}}} \cdot \dfrac{1}{8\sqrt{4}}$

$= \dfrac{1}{\sqrt{1-\dfrac{1}{4}}} \cdot \dfrac{1}{16} = \dfrac{2}{\sqrt{3}} \cdot \dfrac{1}{16} = \dfrac{\sqrt{3}}{24}$

11. $\frac{dx}{dt} = \frac{d}{dt}\left[\tan^{-1} t\right] = \frac{1}{1+t^2}$

$v(2) = \left.\frac{dx}{dt}\right|_{t=2} = \frac{1}{1+2^2} = \frac{1}{5}$

12. $\frac{dx}{dt} = \frac{d}{dt}\left[\tan^{-1}(t^2)\right]$

$= \frac{1}{1+(t^2)^2} \cdot \frac{d}{dt}(t^2)$

$= \frac{2t}{1+t^4}$

$v(1) = \left.\frac{dx}{dt}\right|_{t=1} = \frac{2(1)}{1+1^4} = 1$

13. $\frac{dy}{ds} = \frac{d}{ds}\sec^{-1}(2s+1)$

$= \frac{1}{|2s+1|\sqrt{(2s+1)^2-1}}\frac{d}{ds}(2s+1)$

$= \frac{1}{|2s+1|\sqrt{4s^2+4s}}(2) = \frac{1}{|2s+1|\sqrt{s^2+s}}$

14. $\frac{dy}{ds} = \frac{d}{ds}\sec^{-1} 5s = \frac{1}{|5s|\sqrt{(5s)^2-1}}\frac{d}{ds}(5s) = \frac{1}{|s|\sqrt{25s^2-1}}$

15. $\frac{dy}{dx} = \frac{d}{dx}\csc^{-1}(x^2+1)$

$= -\frac{1}{|x^2+1|\sqrt{(x^2+1)^2-1}}\frac{d}{dx}(x^2+1)$

$= -\frac{2x}{(x^2+1)\sqrt{x^4+2x^2}} = -\frac{2}{(x^2+1)\sqrt{x^2+2}}$

Note that the condition $x > 0$ is required in the last step.

16. $\frac{dy}{dx} = \frac{d}{dx}\csc^{-1}\left(\frac{x}{2}\right) = -\frac{1}{\left|\frac{x}{2}\right|\sqrt{\left(\frac{x}{2}\right)^2-1}}\frac{d}{dx}\left(\frac{x}{2}\right)$

$= -\frac{2}{|x|\sqrt{x^2-4}}$

17. $\frac{dy}{dt} = \frac{d}{dt}\sec^{-1}\left(\frac{1}{t}\right) = \frac{1}{\left|\frac{1}{t}\right|\sqrt{\left(\frac{1}{t}\right)^2-1}}\frac{d}{dt}\left(\frac{1}{t}\right)$

$= \frac{1}{\left|\frac{1}{t}\right|\sqrt{\left(\frac{1}{t}\right)^2-1}}\left(-\frac{1}{t^2}\right) = -\frac{1}{\sqrt{1-t^2}}$

Note that the condition $t > 0$ is required in the last step.

18. $\frac{dy}{dt} = \frac{d}{dt}\cot^{-1}\sqrt{t} = -\frac{1}{1+(\sqrt{t})^2}\frac{d}{dt}\sqrt{t}$

$= -\frac{1}{2\sqrt{t}(t+1)}$

19. $\frac{dy}{dt} = \frac{d}{dt}\cot^{-1}\sqrt{t-1} = -\frac{1}{1+(\sqrt{t-1})^2}\frac{d}{dt}\sqrt{t-1}$

$= -\left(\frac{1}{1+t-1}\right)\left(\frac{1}{2\sqrt{t-1}}\right) = -\frac{1}{2t\sqrt{t-1}}$

20. $\frac{dy}{ds} = \frac{d}{ds}\sqrt{s^2-1} - \frac{d}{ds}\sec^{-1} s$

$= \frac{1}{2\sqrt{s^2-1}}(2s) - \frac{1}{|s|\sqrt{s^2-1}}$

$= \frac{s|s|-1}{|s|\sqrt{s^2-1}}$

21. $\frac{dy}{dx} = \frac{d}{dx}(\tan^{-1}\sqrt{x^2-1}) + \frac{d}{dx}(\csc^{-1} x)$

$= \frac{1}{1+(\sqrt{x^2-1})^2}\frac{d}{dx}(\sqrt{x^2-1}) - \frac{1}{|x|\sqrt{x^2-1}}$

$= \frac{1}{x^2}\frac{1}{2\sqrt{x^2-1}}(2x) - \frac{1}{|x|\sqrt{x^2-1}}$

$= \frac{1}{x\sqrt{x^2-1}} - \frac{1}{|x|\sqrt{x^2-1}}$

$= 0$

Note that the condition $x > 1$ is required in the last step.

22. $\frac{dy}{dx} = \frac{d}{dx}\left(\cot^{-1}\frac{1}{x}\right) - \frac{d}{dx}(\tan^{-1} x)$

$= -\frac{1}{1+\left(\frac{1}{x^2}\right)}\frac{d}{dx}\left(\frac{1}{x}\right) - \frac{1}{1+x^2}$

$= \left(-\frac{1}{1+\frac{1}{x^2}}\right)\left(-\frac{1}{x^2}\right) - \frac{1}{1+x^2}$

$= \frac{1}{x^2+1} - \frac{1}{1+x^2}$

$= 0, x \neq 0$

The condition $x \neq 0$ is required because the original function was undefined when $x = 0$.

23. $y = \sec^{-1} x$

$\frac{dy}{dx} = \frac{1}{|x|\sqrt{x^2-1}}$

$y'(2) = \frac{1}{|2|\sqrt{2^2-1}} = \frac{1}{2\sqrt{3}}$

$y(2) = \sec^{-1}(2) = \cos^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{3}$

$y = \frac{1}{2\sqrt{3}}(x-2) + \frac{\pi}{3}$

or $y = 0.289(x-2) + 1.047$

$y = 0.289x + 0.469$