13. $\frac{dy}{dx}=\frac{d}{dx}(x+\sqrt{x})^{-2}=-2(x+\sqrt{x})^{-3}\frac{d}{dx}(x+\sqrt{x})$

$=-2(x+\sqrt{x})^{-3}\left(1+\frac{1}{2\sqrt{x}}\right)$

14. $\frac{dy}{dx}=\frac{d}{dx}(\csc x+\cot x)^{-1}$

$=-(\csc x+\cot x)^{-2}\frac{d}{dx}(\csc x+\cot x)$

$=-\frac{1}{(\csc x+\cot x)^2}(-\cot x\csc x-\csc^2 x)$

$=\frac{(\csc x)(\cot x+\csc x)}{(\csc x+\cot x)^2}=\frac{\csc x}{\csc x+\cot x}$

15. $\frac{dy}{dx}=\frac{d}{dx}(\sin^{-5}x-\cos^3 x)$

$=(-5\sin^{-6}x)\frac{d}{dx}(\sin x)-(3\cos^2 x)\frac{d}{dx}(\cos x)$

$=-5\sin^{-6}x\cos x+3\cos^2 x\sin x$

16. $\frac{dy}{dx}=\frac{d}{dx}[x^3(2x-5)^4]$

$=(x^3)\frac{d}{dx}(2x-5)^4+(2x-5)^4\frac{d}{dx}(x^3)$

$=(x^3)(4)(2x-5)^3\frac{d}{dx}(2x-5)+(2x-5)^4(3x^2)$

$=(x^3)(4)(2x-5)^3(2)+3x^2(2x-5)^4$

$=8x^3(2x-5)^3+3x^2(2x-5)^4$

$=x^2(2x-5)^3[8x+3(2x-5)]$

$=x^2(2x-5)^3(14x-15)$

17. $\frac{dy}{dx}=\frac{d}{dx}(\sin^3 x\tan 4x)$

$=(\sin^3 x)\frac{d}{dx}(\tan 4x)+(\tan 4x)\frac{d}{dx}(\sin^3 x)$

$=(\sin^3 x)(\sec^2 4x)\frac{d}{dx}(4x)+(\tan 4x)(3\sin^2 x)\frac{d}{dx}(\sin x)$

$=(\sin^3 x)(\sec^2 4x)(4)+(\tan 4x)(3\sin^2 x)(\cos x)$

$=4\sin^3 x\sec^2 4x+3\sin^2 x\cos x\tan 4x$

18. $\frac{dy}{dx}=\frac{d}{dx}(4\sqrt{\sec x+\tan x})$

$=4\cdot\frac{1}{2\sqrt{\sec x+\tan x}}\frac{d}{dx}(\sec x+\tan x)$

$=\frac{2}{\sqrt{\sec x+\tan x}}(\sec x\tan x+\sec^2 x)$

$=2\sec x\frac{\sec x+\tan x}{\sqrt{\sec x+\tan x}}$

$=2\sec x\sqrt{\sec x+\tan x}$

19. $\frac{dy}{dx}=\frac{d}{dx}\left(\frac{3}{\sqrt{2x+1}}\right)$

$=\frac{(\sqrt{2x+1})\frac{d}{dx}(3)-3\frac{d}{dx}(\sqrt{2x+1})}{(\sqrt{2x+1})^2}$

$=\frac{(\sqrt{2x+1})(0)-3\left(\frac{1}{2\sqrt{2x+1}}\right)\frac{d}{dx}(2x+1)}{2x+1}$

$=\frac{-3\left(\frac{1}{2\sqrt{2x+1}}\right)(2)}{2x+1}$

$=-\frac{3}{(2x+1)\sqrt{2x+1}}$

$=-3(2x+1)^{-3/2}$

20. $\frac{dy}{dx}=\frac{d}{dx}\left(\frac{x}{\sqrt{1+x^2}}\right)$

$=\frac{(\sqrt{1+x^2})\frac{d}{dx}(x)-x\frac{d}{dx}(\sqrt{1+x^2})}{(\sqrt{1+x^2})^2}$

$=\frac{(\sqrt{1+x^2})(1)-x\left(\frac{1}{2\sqrt{1+x^2}}\right)\frac{d}{dx}(1+x^2)}{1+x^2}$

$=\frac{\sqrt{1+x^2}-x\left(\frac{1}{2\sqrt{1+x^2}}\right)(2x)}{1+x^2}$

$=\frac{(1+x^2)-x^2}{(1+x^2)(\sqrt{1+x^2})}$

$=(1+x^2)^{-3/2}$

21. The last step here uses the identity 2 sin a cos a = sin 2a.

$\frac{dy}{dx}=\frac{d}{dx}\sin^2(3x-2)$

$=2\sin(3x-2)\frac{d}{dx}\sin(3x-2)$

$=2\sin(3x-2)\cos(3x-2)\frac{d}{dx}(3x-2)$

$=2\sin(3x-2)\cos(3x-2)(3)$

$=6\sin(3x-2)\cos(3x-2)$

$=3\sin(6x-4)$

22. $\frac{dy}{dx}=\frac{d}{dx}(1+\cos 2x)^2=2(1+\cos 2x)\frac{d}{dx}(1+\cos 2x)$

$=2(1+\cos 2x)(-\sin 2x)\frac{d}{dx}(2x)$

$=2(1+\cos 2x)(-\sin 2x)(2)$

$=-4(1+\cos 2x)(\sin 2x)$

23. $\frac{dy}{dx} = \frac{d}{dx}(1+\cos^2 7x)^3$

$= 3(1+\cos^2 7x)^2 \frac{d}{dx}(1+\cos^2 7x)$

$= 3(1+\cos^2 7x)^2 (2\cos 7x)\frac{d}{dx}(\cos 7x)$

$= 3(1+\cos^2 7x)^2 (2\cos 7x)(-\sin 7x)\frac{d}{dx}(7x)$

$= 3(1+\cos^2 7x)^2 (2\cos 7x)(-\sin 7x)(7)$

$= -42(1+\cos^2 7x)^2 \cos 7x \sin 7x$

24. $\frac{dy}{dx} = \frac{d}{dx}(\sqrt{\tan 5x}) = \frac{1}{2\sqrt{\tan 5x}}\frac{d}{dx}\tan 5x$

$= \frac{1}{2\sqrt{\tan 5x}}(\sec^2 5x)\frac{d}{dx}(5x)$

$= \frac{1}{2\sqrt{\tan 5x}}(\sec^2 5x)(5)$

$= \frac{5\sec^2 5x}{2\sqrt{\tan 5x}}$ or $\frac{5}{2}(\tan 5x)^{-1/2}\sec^2 5x$

25. $\frac{dr}{d\theta} = \frac{d}{d\theta}\tan(2-\theta) = \sec^2(2-\theta)\frac{d}{d\theta}(2-\theta)$

$= \sec^2(2-\theta)(-1) = -\sec^2(2-\theta)$

26. $\frac{dr}{d\theta} = \frac{d}{d\theta}(\sec 2\theta \ \tan 2\theta)$

$= (\sec 2\theta)\frac{d}{d\theta}(\tan 2\theta) + (\tan 2\theta)\frac{d}{d\theta}(\sec 2\theta)$

$= (\sec 2\theta)(\sec^2 2\theta)\frac{d}{d\theta}(2\theta) + (\tan 2\theta)(\sec 2\theta \tan 2\theta)\frac{d}{d\theta}(2\theta)$

$= 2\sec^3 2\theta + 2\sec 2\theta \tan^2 2\theta$

27. $\frac{dr}{d\theta} = \frac{d}{d\theta}\sqrt{\theta \sin\theta} = \frac{1}{2\sqrt{\theta\sin\theta}}\frac{d}{d\theta}(\theta\sin\theta)$

$= \frac{1}{2\sqrt{\theta\sin\theta}}\left[\theta\frac{d}{d\theta}(\sin\theta) + (\sin\theta)\frac{d}{d\theta}(\theta)\right]$

$= \frac{1}{2\sqrt{\theta\sin\theta}}(\theta\cos\theta + \sin\theta)$

$= \frac{\theta\cos\theta + \sin\theta}{2\sqrt{\theta\sin\theta}}$

28. $\frac{dr}{d\theta} = \frac{d}{d\theta}(2\theta\sqrt{\sec\theta})$

$= (2\theta)\frac{d}{d\theta}(\sqrt{\sec\theta}) + (\sqrt{\sec\theta})\frac{d}{d\theta}(2\theta)$

$= (2\theta)\left(\frac{1}{2\sqrt{\sec\theta}}\right)\frac{d}{d\theta}(\sec\theta) + 2\sqrt{\sec\theta}$

$= (2\theta)\left(\frac{1}{2\sqrt{\sec\theta}}\right)(\sec\theta\tan\theta) + 2\sqrt{\sec\theta}$

$= \theta(\sqrt{\sec\theta})(\tan\theta) + 2\sqrt{\sec\theta}$

$= \sqrt{\sec\theta}(\theta\tan\theta + 2)$

29. $y' = \frac{d}{dx}\tan x = \sec^2 x$

$y'' = \frac{d}{dx}\sec^2 x = (2\sec x)\frac{d}{dx}(\sec x)$

$= (2\sec x)(\sec x \tan x)$

$= 2\sec^2 x \tan x$

30. $y' = \frac{d}{dx}\cot x = -\csc^2 x$

$y'' = \frac{d}{dx}(-\csc^2 x) = (-2\csc x)\frac{d}{dx}(\csc x)$

$= (-2\csc x)(-\csc x \cot x)$

$= 2\csc^2 x \cot x$

31. $y' = \frac{d}{dx}\cot(3x-1) = -\csc^2(3x-1)\frac{d}{dx}(3x-1)$

$= -3\csc^2(3x-1)$

$y'' = \frac{d}{dx}[-3\csc^2(3x-1)]$

$= -3[2\csc(3x-1)]\frac{d}{dx}\csc(3x-1)$

$= -3[2\csc(3x-1)] \bullet$

$[-\csc(3x-1)\cot(3x-1)]\frac{d}{dx}(3x-1)$

$= -3[2\csc(3x-1)][-\csc(3x-1)\cot(3x-1)](3)$

$= 18\csc^2(3x-1)\cot(3x-1)$

32. $y' = \frac{d}{dx}\left[9\tan\left(\frac{x}{3}\right)\right] = 9\sec^2\left(\frac{x}{3}\right)\frac{d}{dx}\left(\frac{x}{3}\right)$

$= 3\sec^2\left(\frac{x}{3}\right)$

$y'' = \frac{d}{dx}\left[3\sec^2\left(\frac{x}{3}\right)\right] = 3\left[2\sec\left(\frac{x}{3}\right)\right]\frac{d}{dx}\sec\left(\frac{x}{3}\right)$

$= 6\left[\sec\left(\frac{x}{3}\right)\right]\left[\sec\left(\frac{x}{3}\right)\tan\left(\frac{x}{3}\right)\right]\frac{d}{dx}\left(\frac{x}{3}\right)$

$= 2\sec^2\left(\frac{x}{3}\right)\tan\left(\frac{x}{3}\right)$

33. $f'(u) = \frac{d}{du}(u^5+1) = 5u^4$

$g'(x) = \frac{d}{dx}(\sqrt{x}) = \frac{1}{2\sqrt{x}}$

$(f \circ g)'(1) = f'(g(1))g'(1) = f'(1)g'(1) = (5)\left(\frac{1}{2}\right) = \frac{5}{2}$

34. $f'(u) = \frac{d}{du}(1-u^{-1}) = u^{-2} = \frac{1}{u^2}$

$g'(x) = \frac{d}{dx}(1-x)^{-1} = -(1-x)^{-2}\frac{d}{dx}(1-x)$

$= -(1-x)^{-2}(-1) = \frac{1}{(1-x)^2}$

$(f \circ g)'(-1) = f'(g(-1))g'(-1) = f'\left(\frac{1}{2}\right)g'(-1)$

$= (4)\left(\frac{1}{4}\right) = 1$

35. $f'(u) = \frac{d}{du}\left(\cot\frac{\pi u}{10}\right) = -\csc^2\left(\frac{\pi u}{10}\right)\frac{d}{du}\left(\frac{\pi u}{10}\right)$

$= -\frac{\pi}{10}\csc^2\left(\frac{\pi u}{10}\right)$

$g'(x) = \frac{d}{dx}(5\sqrt{x}) = \frac{5}{2\sqrt{x}}$

$(f \circ g)'(1) = f'(g(1))g'(1) = f'(5)g'(1)$

$= -\frac{\pi}{10}\left[\csc^2\left(\frac{\pi}{2}\right)\right]\left(\frac{5}{2}\right)$

$= -\frac{\pi}{10}(1)\left(\frac{5}{2}\right) = -\frac{\pi}{4}$

36. $f'(u) = \frac{d}{du}\left[u + (\cos u)^{-2}\right]$

$= 1 - 2(\cos u)^{-3}\frac{d}{du}\cos u$

$= 1 + \frac{2\sin u}{\cos^3 u}$

$g'(x) = \frac{d}{dx}(\pi x) = \pi$

$(f \circ g)'\left(\frac{1}{4}\right) = f'(g(\frac{1}{4}))g'\left(\frac{1}{4}\right)$

$= f'\left(\frac{\pi}{4}\right)g'\left(\frac{1}{4}\right)$

$= \left(1 + \frac{\frac{2}{\sqrt{2}}}{\left(\frac{1}{\sqrt{2}}\right)^3}\right)(\pi)$

$= 5\pi$

37. $f'(u) = \frac{d}{du}\frac{2u}{u^2+1} = \frac{(u^2+1)\frac{d}{du}(2u) - (2u)\frac{d}{du}(u^2+1)}{(u^2+1)^2}$

$= \frac{(u^2+1)(2) - (2u)(2u)}{(u^2+1)^2} = \frac{-2u^2+2}{(u^2+1)^2}$

$g'(x) = \frac{d}{dx}(10x^2 + x + 1) = 20x + 1$

$(f \circ g)'(0) = f'(g(0))g'(0) = f'(1)g'(0) = (0)(1) = 0$

38. $f'(u) = \frac{d}{du}\left(\frac{u-1}{u+1}\right)^2 = 2\left(\frac{u-1}{u+1}\right)\frac{d}{du}\left(\frac{u-1}{u+1}\right)$

$= 2\left(\frac{u-1}{u+1}\right)\frac{(u+1)\frac{d}{du}(u-1) - (u-1)\frac{d}{du}(u+1)}{(u+1)^2}$

$= 2\left(\frac{u-1}{u+1}\right)\frac{(u+1)-(u-1)}{(u+1)^2} = \frac{4(u-1)}{(u+1)^3}$

$g'(x) = \frac{d}{dx}(x^{-2} - 1) = -2x^{-3}$

$(f \circ g)'(-1) = f'(g(-1))g'(-1)$

$= f'(0)g'(-1)$

$= (-4)(2) = -8$

39. (a) $\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$

$= \frac{d}{du}(\cos u)\frac{d}{dx}(6x+2)$

$= (-\sin u)(6)$

$= -6\sin u$

$= -6\sin(6x+2)$

(b) $\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$

$= \frac{d}{du}(\cos 2u)\frac{d}{dx}(3x+1)$

$= (-\sin 2u)(2) \cdot (3)$

$= -6\sin 2u$

$= -6\sin(6x+2)$

40. (a) $\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$

$= \frac{d}{du}\sin(u+1)\frac{d}{dx}(x^2)$

$= \cos(u+1)(1) \cdot 2x$

$= 2x\cos(u+1)$

$= 2x\cos(x^2+1)$

(b) $\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$

$= \frac{d}{du}(\sin u)\frac{d}{dx}(x^2+1)$

$= (\cos u)(2x)$

$= 2x\cos u$

$= 2x\cos(x^2+1)$

41. $\frac{dx}{dt} = \frac{d}{dt}(2\cos t) = -2\sin t$

$\frac{dy}{dt} = \frac{d}{dt}(2\sin t) = 2\cos t$

$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{2\cos t}{-2\sin t} = -\cot t$

This line passes through $\left(2\cos\frac{\pi}{4},\ 2\sin\frac{\pi}{4}\right) = (\sqrt{2},\ \sqrt{2})$ and has slope $-\cot\frac{\pi}{4} = -1$. Its equation is $y = -(x-\sqrt{2}) + \sqrt{2}$, or $y = -x + 2\sqrt{2}$.

42. $\frac{dx}{dt} = \frac{d}{dt}(\sin 2\pi t) = (\cos 2\pi t)\frac{d}{dt}(2\pi t) = 2\pi\cos 2\pi t$

$\frac{dy}{dt} = \frac{d}{dt}(\cos 2\pi t) = (-\sin 2\pi t)\frac{d}{dt}(2\pi t) = -2\pi\sin 2\pi t$

$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{-2\pi\sin 2\pi t}{2\pi\cos 2\pi t} = -\tan 2\pi t$

The line passes through $\left(\sin\frac{2\pi}{-6},\ \cos\frac{2\pi}{-6}\right) = \left(-\frac{\sqrt{3}}{2},\ \frac{1}{2}\right)$ and has slope $-\tan\frac{2\pi}{-6} = \sqrt{3}$. Its equation is $y = \sqrt{3}\left(x + \frac{\sqrt{3}}{2}\right) + \frac{1}{2}$, or $y = \sqrt{3}x + 2$.

43. $\frac{dx}{dt} = \frac{d}{dt}(\sec^2 t - 1) = (2\sec t)\frac{d}{dt}(\sec t)$
$= (2\sec t)(\sec t\tan t)$
$= 2\sec^2 t\tan t$

$\frac{dy}{dt} = \frac{d}{dt}\tan t = \sec^2 t$

$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{\sec^2 t}{2\sec^2 t\tan t} = \frac{1}{2}\cot t.$

The line passes through $\left(\sec^2\left(-\frac{\pi}{4}\right) - 1,\ \tan\left(-\frac{\pi}{4}\right)\right) = (1,\ -1)$ and has slope $\frac{1}{2}\cot\left(-\frac{\pi}{4}\right) = -\frac{1}{2}$. Its equation is $y = -\frac{1}{2}(x-1) - 1$, or $y = -\frac{1}{2}x - \frac{1}{2}$.

44. $\frac{dx}{dt} = \frac{d}{dt}\sec t = \sec t\tan t$

$\frac{dy}{dt} = \frac{d}{dt}\tan t = \sec^2 t$

$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{\sec^2 t}{\sec t\tan t} = \frac{\sec t}{\tan t} = \frac{1}{\sin t} = \csc t$

The line passes through $\left(\sec\frac{\pi}{6},\ \tan\frac{\pi}{6}\right) = \left(\frac{2}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$ and has slope $\csc\frac{\pi}{6} = 2$. Its equation is $y = 2\left(x - \frac{2}{\sqrt{3}}\right) + \frac{1}{\sqrt{3}}$, or $y = 2x - \sqrt{3}$.

45. $\frac{dx}{dt} = \frac{d}{dt}t = 1$

$\frac{dy}{dt} = \frac{d}{dt}\sqrt{t} = \frac{1}{2\sqrt{t}}$

$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{1/(2\sqrt{t})}{1} = \frac{1}{2\sqrt{t}}$

The line passes through $\left(\frac{1}{4}, \sqrt{\frac{1}{4}}\right) = \left(\frac{1}{4}, \frac{1}{2}\right)$ and has slope $\frac{1}{2\sqrt{\frac{1}{4}}} = 1$. Its equation is $y = 1\left(x - \frac{1}{4}\right) + \frac{1}{2}$, or $y = x + \frac{1}{4}$.

46. $\frac{dx}{dt} = \frac{d}{dt}(2t^2 + 3) = 4t$

$\frac{dy}{dt} = \frac{d}{dt}(t^4) = 4t^3$

$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{4t^3}{4t} = t^2$

The line passes through $(2(-1)^2 + 3, (-1)^4) = (5, 1)$ and has slope $(-1)^2 = 1$. Its equation is $y = 1(x-5) + 1$, or $y = x - 4$.

47. $\frac{dx}{dt} = \frac{d}{dt}(t - \sin t) = 1 - \cos t$

$\frac{dy}{dt} = \frac{d}{dt}(1 - \cos t) = \sin t$

$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{\sin t}{1 - \cos t}$

The line passes through

$\left(\frac{\pi}{3} - \sin\frac{\pi}{3},\ 1 - \cos\frac{\pi}{3}\right) = \left(\frac{\pi}{3} - \frac{\sqrt{3}}{2},\ \frac{1}{2}\right)$ and has slope

$\frac{\sin\left(\frac{\pi}{3}\right)}{1 - \cos\left(\frac{\pi}{3}\right)} = \sqrt{3}$. Its equation is

$y = \sqrt{3}\left(x - \frac{\pi}{3} + \frac{\sqrt{3}}{2}\right) + \frac{1}{2}$, or

$y = \sqrt{3}x + 2 - \frac{\pi}{\sqrt{3}}$.

48. $\frac{dx}{dt} = \frac{d}{dt}\cos t = -\sin t$

$\frac{dy}{dt} = \frac{d}{dt}(1 + \sin t) = \cos t$

$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{\cos t}{-\sin t} = -\cot t$

The line passes through $\left(\cos\frac{\pi}{2},\ 1 + \sin\frac{\pi}{2}\right) = (0, 2)$ and has slope $-\cot\left(\frac{\pi}{2}\right) = 0$. Its equation is $y = 2$.

49. (a) $\frac{dx}{dt} = \frac{d}{dt}(t^2 + t) = 2t + 1$

$\frac{dy}{dt} = \frac{d}{dt}\sin t = \cos t$

$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{\cos t}{2t + 1}$

(b) $\frac{d}{dt}\left(\frac{dy}{dx}\right) = \frac{d}{dt}\frac{\cos t}{2t + 1}$

$= \frac{(2t+1)\frac{d}{dt}(\cos t) - (\cos t)\frac{d}{dt}(2t+1)}{(2t+1)^2}$

$= \frac{(2t+1)(-\sin t) - (\cos t)(2)}{(2t+1)^2}$

$= -\frac{(2t+1)(\sin t) + 2\cos t}{(2t+1)^2}$

(c) Let $u = \frac{dy}{dx}$.

Then $\frac{du}{dt} = \frac{du}{dx}\frac{dx}{dt}$, so $\frac{du}{dx} = \frac{du}{dt} \div \frac{dx}{dt}$. Therefore,

$\frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{dt}\left(\frac{dy}{dx}\right) \div \frac{dx}{dt}$

$= -\frac{(2t+1)(\sin t) + 2\cos t}{(2t+1)^2} \div (2t+1)$

$= -\frac{(2t+1)(\sin t) + 2\cos t}{(2t+1)^3}$

(d) The expression in part (c).

50. Since the radius passes through $(0, 0)$ and $(2\cos t, 2\sin t)$, it has slope given by $\tan t$. But the slope of the tangent line is

$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{2\cos t}{-2\sin t} = -\cot t$, which is the negative reciprocal of $\tan t$. This means that the radius and the tangent line are perpendicular. (The preceding argument breaks down when $t = \frac{k\pi}{2}$, where k is an integer. At these values, either the radius is horizontal and the tangent line is vertical or the radius is vertical and the tangent line is horizontal, so the result still holds.)

51. $\frac{ds}{dt} = \frac{ds}{d\theta}\frac{d\theta}{dt} = \frac{d}{d\theta}(\cos\theta)\frac{d\theta}{dt}$

$= (-\sin\theta)\left(\frac{d\theta}{dt}\right)$

When $\theta = \frac{3\pi}{2}$ and $\frac{d\theta}{dt} = 5$, $\frac{ds}{dt} = \left(-\sin\frac{3\pi}{2}\right)(5) = 5$.

52. $\frac{dy}{dt} = \frac{dy}{dx}\frac{dx}{dt} = \frac{d}{dx}(x^2 + 7x - 5)\frac{dx}{dt}$

$= (2x + 7)\left(\frac{dx}{dt}\right)$

When $x = 1$ and $\frac{dx}{dt} = \frac{1}{3}$, $\frac{dy}{dt} = [2(1) + 7]\left(\frac{1}{3}\right) = 3$.

53. $\frac{dy}{dx} = \frac{d}{dx}\sin\frac{x}{2} = \left(\cos\frac{x}{2}\right)\frac{d}{dx}\left(\frac{x}{2}\right) = \frac{1}{2}\cos\frac{x}{2}$

Since the range of the function $f(x) = \frac{1}{2}\cos\frac{x}{2}$ is $\left[-\frac{1}{2}, \frac{1}{2}\right]$, the largest possible value of $\frac{dy}{dx}$ is $\frac{1}{2}$.

54. $\frac{dy}{dx} = \frac{d}{dx}(\sin mx) = (\cos mx)\frac{d}{dx}(mx) = m\cos mx$

The desired line has slope $y'(0) = m\cos 0 = m$ and passes through $(0, 0)$, so its equation is $y = mx$.

55. $\frac{dy}{dx} = \frac{d}{dx}\left(2\tan\frac{\pi x}{4}\right) = \left(2\sec^2\frac{\pi x}{4}\right)\frac{d}{dx}\left(\frac{\pi x}{4}\right)$

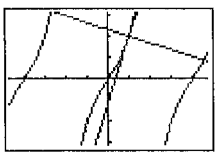
$= \frac{\pi}{2}\sec^2\left(\frac{\pi x}{4}\right)$

$y'(1) = \frac{\pi}{2}\sec^2\left(\frac{\pi}{4}\right) = \frac{\pi}{2}(\sqrt{2})^2 = \pi.$

The tangent line has slope π and passes through $\left(1, 2\tan\frac{\pi}{4}\right) = (1, 2)$. Its equation is $y = \pi(x-1)+2$, or $y = \pi x - \pi + 2$.

The normal line has slope $-\frac{1}{\pi}$ and passes through (1, 2). Its equation is $y = -\frac{1}{\pi}(x-1)+2$, or $y = -\frac{1}{\pi}x + \frac{1}{\pi} + 2.$

Graphical support:



[–4.7, 4.7] by [–3.1, 3.1]

56. (a) $\frac{d}{dx}[2f(x)] = 2f'(x)$

At $x = 2$, the derivative is $2f'(2) = 2\left(\frac{1}{3}\right) = \frac{2}{3}$.

(b) $\frac{d}{dx}[f(x)+g(x)] = f'(x)+g'(x)$

At $x = 3$, the derivative is $f'(3)+g'(3) = 2\pi + 5$.

(c) $\frac{d}{dx}[f(x)\cdot g(x)] = f(x)g'(x)+g(x)f'(x)$

At $x = 3$, the derivative is

$f(3)g'(3)+g(3)f'(3) = (3)(5)+(-4)(2\pi)$
$= 15 - 8\pi.$

(d) $\frac{d}{dx}\frac{f(x)}{g(x)} = \frac{g(x)f'(x)-f(x)g'(x)}{[g(x)]^2}$

At $x = 2$, the derivative is

$\frac{g(2)f'(2)-f(2)g'(2)}{[g(2)]^2} = \frac{(2)\left(\frac{1}{3}\right)-(8)(-3)}{(2)^2}$

$= \frac{\frac{74}{3}}{4} = \frac{37}{6}.$

(e) $\frac{d}{dx}f(g(x)) = f'(g(x))g'(x)$

At $x = 2$, the derivative is

$f'(g(2))g'(2) = f'(2)g'(2) = \left(\frac{1}{3}\right)(-3) = -1.$

(f) $\frac{d}{dx}\sqrt{f(x)} = \frac{1}{2\sqrt{f(x)}}\frac{d}{dx}f(x) = \frac{f'(x)}{2\sqrt{f(x)}}$

At $x = 2$, the derivative is

$\frac{f'(2)}{2\sqrt{f(2)}} = \frac{\frac{1}{3}}{2\sqrt{8}} = \frac{1}{6(2\sqrt{2})} = \frac{1}{12\sqrt{2}}.$

(g) $\frac{d}{dx}\frac{1}{g^2(x)} = \frac{d}{dx}[g(x)]^{-2} = -2[g(x)]^{-3}\frac{d}{dx}g(x) = -\frac{2g'(x)}{[g(x)]^3}$

At $x = 3$, the derivative is

$-\frac{2g'(3)}{[g(3)]^3} = -\frac{2(5)}{(-4)^3} = -\frac{10}{-64} = \frac{5}{32}.$

(h) $\frac{d}{dx}\sqrt{f^2(x)+g^2(x)} = \frac{1}{2\sqrt{f^2(x)+g^2(x)}}\frac{d}{dx}[f^2(x)+g^2(x)]$

$= \frac{1}{2\sqrt{f^2(x)+g^2(x)}}[2f(x)\frac{d}{dx}f(x)+2g(x)\frac{d}{dx}g(x)]$

$= \frac{f(x)f'(x)+g(x)g'(x)}{\sqrt{f^2(x)+g^2(x)}}$

At $x = 2$, the derivative is

$\frac{f(2)f'(2)+g(2)g'(2)}{\sqrt{f^2(2)+g^2(2)}} = \frac{(8)\left(\frac{1}{3}\right)+(2)(-3)}{\sqrt{8^2+2^2}}$

$= \frac{-\frac{10}{3}}{\sqrt{68}} = -\frac{\frac{10}{3}}{2\sqrt{17}} = -\frac{5}{3\sqrt{17}}$

57. $\frac{d}{dx}\cos(x°) = \frac{d}{dx}\cos\left(\frac{\pi x}{180}\right) = -\frac{\pi}{180}\sin\left(\frac{\pi x}{180}\right) = -\frac{\pi}{180}\sin(x°)$

58. (a) $\frac{d}{dx}[5f(x)-g(x)] = 5f'(x)-g'(x)$

At $x = 1$, the derivative is

$5f'(1)-g'(1) = 5\left(-\frac{1}{3}\right)-\left(-\frac{8}{3}\right) = 1.$

(b) $\frac{d}{dx}\left(f(x)g^3(x)\right) = f(x)\frac{d}{dx}g^3(x)+g^3(x)\frac{d}{dx}f(x)$

$= f(x)[3g^2(x)]\frac{d}{dx}g(x)+g^3(x)f'(x)$

$= 3f(x)g^2(x)g'(x)+g^3(x)f'(x)$

At $x = 0$, the derivative is $3f(0)g^2(0)g'(0)+g^3(0)f'(0)$

$= 3(1)(1)^2\left(\frac{1}{3}\right)+(1)^3(5) = 6.$

58. Continued

(c) $$\frac{d}{dx}\left(\frac{f(x)}{g(x)+1}\right)=\frac{[g(x)+1]\frac{d}{dx}f(x)-f(x)\frac{d}{dx}[g(x)+1]}{[g(x)+1]^2}$$

$$=\frac{[g(x)+1]f'(x)-f(x)g'(x)}{[g(x)+1]^2}$$

At $x=1$, the derivative is

$$\frac{[g(1)+1]f'(1)-f(1)g'(1)}{[g(1)+1]^2}=\frac{(-4+1)\left(-\frac{1}{3}\right)-(3)\left(-\frac{8}{3}\right)}{(-4+1)^2}$$

$$=\frac{9}{9}=1.$$

(d) $\frac{d}{dx}f(g(x))=f'(g(x))g'(x)$

At $x=0$, the derivative is

$$f'(g(0))g'(0)=f'(1)g'(0)=\left(-\frac{1}{3}\right)\left(\frac{1}{3}\right)=-\frac{1}{9}.$$

(e) $\frac{d}{dx}g(f(x))=g'(f(x))f'(x)$

At $x=0$, the derivative is

$$g'(f(0))f'(0)=g'(1)f'(0)=\left(-\frac{8}{3}\right)(5)=-\frac{40}{3}$$

(f) $$\frac{d}{dx}[g(x)+f(x)]^{-2}=-2[g(x)+f(x)]^{-3}\frac{d}{dx}[g(x)+f(x)]$$

$$=-\frac{2[g'(x)+f'(x)]}{[g(x)+f(x)]^3}$$

At $x=1$, the derivative is

$$-\frac{2[g'(1)+f'(1)]}{[g(1)+f(1)]^3}=-\frac{2\left(-\frac{8}{3}-\frac{1}{3}\right)}{(-4+3)^3}=-\frac{-6}{-1}=-6.$$

(g) $$\frac{d}{dx}[f(x+g(x))]=f'(x+g(x))\frac{d}{dx}[x+g(x)]$$

$$=f'(x+g(x))(1+g'(x))$$

At $x=0$, the derivative is

$$f'(0+g(0))(1+g'(0))=f'(0+1)\left(1+\frac{1}{3}\right)$$

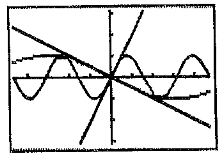
$$=f'(1)\left(\frac{4}{3}\right)$$

$$=\left(-\frac{1}{3}\right)\left(\frac{4}{3}\right)=-\frac{4}{9}.$$

59. For $y=\sin 2x$, $y'=(\cos 2x)\frac{d}{dx}(2x)=2\cos 2x$ and the slope at the origin is 2.

For $y=-\sin\frac{x}{2}$, $y'=\left(-\cos\frac{x}{2}\right)\frac{d}{dx}\left(\frac{x}{2}\right)=-\frac{1}{2}\cos\frac{x}{2}$ and the slope at the origin is $-\frac{1}{2}$. Since the slopes of the two tangent lines are 2 and $-\frac{1}{2}$, the lines are perpendicular and the curves are orthogonal.

A graph of the two curves along with the tangents $y=2x$ and $y=-\frac{1}{2}x$ is shown.



[−4.7, 4.7] by [−3.1, 3.1]

60. Because the symbols $\frac{dy}{dx}$, $\frac{dy}{du}$, and $\frac{du}{dx}$ are not fractions. The individual symbols dy, dx, and du do not have numerical values.

61. Velocity: $s'(t)=-2\pi bA\sin(2\pi bt)$

acceleration: $s''(t)=-4\pi^2b^2A\cos(2\pi bt)$

jerk: $s'''(t)=8\pi^3b^3A\sin(2\pi bt)$

The velocity, amplitude, and jerk are proportional to b, b^2, and b^3 respectively. If the frequency b is doubled, then the amplitude of the velocity is doubled, the amplitude of the acceleration is quadrupled, and the amplitude of the jerk is multiplied by 8.

62. (a) $$y'(t)=\frac{d}{dt}37\sin\left[\frac{2\pi}{365}(x-101)\right]+\frac{d}{dt}(25)$$

$$=37\cos\left[\frac{2\pi}{365}(x-101)\right]\cdot\frac{d}{dx}\left[\frac{2\pi}{365}(x-101)\right]+0$$

$$=37\cos\left[\frac{2\pi}{365}(x-101)\right]\cdot\frac{2\pi}{365}$$

$$=\frac{74\pi}{365}\cos\left[\frac{2\pi}{365}(x-101)\right]$$

Since $\cos u$ is greatest when $u=0$, $\pm 2\pi$, and so on, $y'(t)$ is greatest when $\frac{2\pi}{365}(x-101)=0$, or $x=101$. The temperature is increasing the fastest on day 101 (April 11).

(b) The rate of increase is

$$y'(101)=\frac{74\pi}{365}\approx 0.637 \text{ degrees per day.}$$

63. Velocity: $$s'(t)=\frac{d}{dt}\sqrt{1+4t}=\frac{1}{2\sqrt{1+4t}}\frac{d}{dt}(1+4t)$$

$$=\frac{4}{2\sqrt{1+4t}}=\frac{2}{\sqrt{1+4t}}$$

At $t=6$, the velocity is $\frac{2}{\sqrt{1+4(6)}}=\frac{2}{5}$ m/sec

63. Continued

Acceleration: $s''(t) = \frac{d}{dt}\frac{2}{\sqrt{1+4t}}$

$$= \frac{(\sqrt{1+4t})\frac{d}{dt}(2) - 2\frac{d}{dt}\sqrt{1+4t}}{(\sqrt{1+4t})^2}$$

$$= \frac{-2\left(\frac{1}{2\sqrt{1+4t}}\right)\frac{d}{dt}(1+4t)}{1+4t}$$

$$= \frac{\frac{-4}{\sqrt{1+4t}}}{1+4t} = -\frac{4}{(1+4t)^{3/2}}$$

At $t = 6$, the acceleration is $-\frac{4}{[1+4(6)]^{3/2}} = -\frac{4}{125}$ m/sec^2

64. Acceleration $= \frac{dv}{dt} = \frac{dv}{ds}\frac{ds}{dt} = \left(\frac{dv}{ds}\right)(v) = \left[\frac{d}{ds}(k\sqrt{s})\right](k\sqrt{s})$

$$= \left(\frac{k}{2\sqrt{s}}\right)(k\sqrt{s}) = \frac{k^2}{2},$$ a constant.

65. Note that this exercise concerns itself with the slowing down caused by the earth's atmosphere, *not* the acceleration caused by gravity.

Given: $v = \frac{k}{\sqrt{s}}$

Acceleration $= \frac{dv}{dt} = \frac{dv}{ds}\frac{ds}{dt} = \left(\frac{dv}{ds}\right)(v) = (v)\left(\frac{dv}{ds}\right)$

$$= \left(\frac{k}{\sqrt{s}}\right)\frac{d}{ds}\frac{k}{\sqrt{s}}$$

$$= \left(\frac{k}{\sqrt{s}}\right)\left(\frac{\sqrt{s}\frac{d}{ds}(k) - k\frac{d}{ds}\sqrt{s}}{(\sqrt{s})^2}\right)$$

$$= \left(\frac{k}{\sqrt{s}}\right)\left(\frac{\frac{-k}{(2\sqrt{s})}}{s}\right)$$

$$= -\frac{k^2}{2s^2}, s \ge 0$$

Thus, the acceleration is inversely proportional to s^2.

66. Acceleration $= \frac{dv}{dt} = \frac{df(x)}{dt} = \frac{df(x)}{dx}\frac{dx}{dt} = f'(x)f(x)$

67. $\frac{dT}{du} = \frac{dT}{dL}\frac{dL}{du} = \left(\frac{d}{dL}2\pi\sqrt{\frac{L}{g}}\right)(kL)$

$$= \left(2\pi\frac{1}{2\sqrt{\frac{L}{g}}}\right)\left(\frac{d}{dL}\frac{L}{g}\right)(kL)$$

$$= \left(\frac{\pi}{\sqrt{\frac{L}{g}}}\right)\left(\frac{1}{g}\right)(kL) = k\pi\sqrt{\frac{L}{g}} = \frac{kT}{2}$$

68. No, this does not contradict the Chain Rule. The Chain Rule states that if two functions are differentiable at the appropriate points, then their composite must also be differentiable. It does not say: If a composite is differentiable, then the functions which make up the composite must all be differentiable.

69. Yes. Note that $\frac{d}{dx}f(g(x)) = f'(g(x))g'(x)$. If the graph of $y = f(g(x))$ has a horizontal tangent at $x = 1$, then $f'(g(1))g'(1) = 0$, so either $g'(1) = 0$ or $f'(g(1)) = 0$. This means that either the graph of $y = g(x)$ has a horizontal tangent at $x = 1$, or the graph of $y = f(u)$ has a horizontal tangent at $u = g(1)$.

70. False. See example 8.

71. False. $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{3\cos t}{-3\sin t} = -\cot(t)$

$$\left.\frac{dy}{dx}\right|_{t=\pi/4} = -\cot\left(\frac{\pi}{4}\right) = -1$$

normal slope: $m_2 = -\frac{1}{m_1} = -\frac{1}{-1} = 1$

The slope of the normal is +1.

72. E. $\frac{dy}{dx} = \frac{d}{dx}\tan(4x)$

$y = \tan u \qquad u = 4x$

$\frac{dy}{du} = \sec^2 u \qquad \frac{du}{dx} = 4$

$\frac{dy}{dx} = 4\sec^2(4x)$

73. C. $\frac{dy}{dx} = \frac{d}{dx}\cos^2(x^3 + x^2)$

$y = \cos^2 u \qquad u = x^3 + x^2$

$\frac{dy}{du} = -2\sin u\cos u \qquad \frac{du}{dx} = 3x^2 + 2x$

$\frac{dy}{dx} = -2(3x^2 + 2x)\cos(x^3 + x^2)\sin(x^3 + x^2)$

74. A. $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$

$\frac{dy}{dt} = \frac{d}{dt}(-1 + \sin t) = \cos t$

$\frac{dx}{dt} = \frac{d}{dt}(t - \cos t) = 1 + \sin t$

$\frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{\cos t}{1 + \sin t}$

$\left.\frac{dy}{dx}\right|_{t=0} = \frac{\cos 0}{1 + \sin 0} = 1$

$x(0) = 0 - \cos 0 = -1$

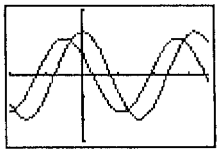
$y(0) = -1 + \sin 0 = -1$

$y = 1(x - (-1)) + (-1) = x$

75. B. See problem 74.

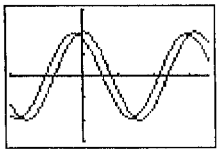
$$\frac{dy}{dx} = \frac{\cos t}{1+\sin t} = 0$$
$$\cos t = 0$$
$$t = \frac{\pi}{2}$$

76. For $h = 1$:



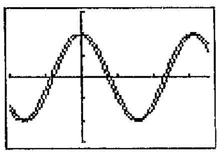
[–2, 3.5] by [–3, 3]

For $h = 0.5$:



[–2, 3.5] by [–3, 3]

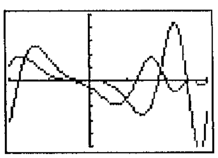
For $h = 0.2$:



[–2, 3.5] by [–3, 3]

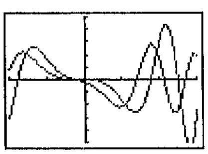
As $h \to 0$, the second curve (the difference quotient) approaches the first ($y = 2\cos 2x$). This is because $2\cos 2x$ is the derivative of $\sin 2x$, and the second curve is the difference quotient used to define the derivative of $\sin 2x$. As $h \to 0$, the difference quotient expression should be approaching the derivative.

77. For $h = 1$:



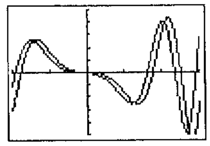
[–2, 3] by [–5, 5]

For $h = 0.3$:



[–2, 3] by [–5, 5]

For $h = 0.3$:



[–2, 3] by [–5, 5]

As $h \to 0$, the second curve (the difference quotient) approaches the first ($y = -2x\sin(x^2)$). This is because $-2x\sin(x^2)$ is the derivative of $\cos(x^2)$, and the second curve is the difference quotient used to define the derivative of $\cos(x^2)$. As $h \to 0$, the difference quotient expression should be approaching the derivative.

78. **(a)** Let $f(x) = |x| = \begin{cases} -x & x < 0 \\ x & x \ge 0 \end{cases}$

Then $f'(x) = \frac{x}{|x|} = \begin{cases} -1 & x < 0 \\ 1 & x > 0 \end{cases}$

and $\frac{d}{dx}|u| = \frac{d}{dx}f(u) = f'(u)\frac{du}{dx} = \frac{u}{|u|}\frac{du}{dx}$

(b) $f'(x) = \left[\frac{d}{dx}(x^2-9)\right] \cdot \frac{x^2-9}{|x^2-9|} = \frac{(2x)(x^2-9)}{|x^2-9|}$

$$\begin{aligned} g'(x) &= \frac{d}{dx}(|x|\sin x) \\ &= |x|\frac{d}{dx}(\sin x) + (\sin x)\frac{d}{dx}|x| \\ &= |x|\cos x + \frac{x\sin x}{|x|} \end{aligned}$$

Note: The expression for $g'(x)$ above is undefined at $x = 0$, but actually

$$g'(0) = \lim_{h\to 0}\frac{g(0+h)-g(0)}{h} = \lim_{h\to 0}\frac{|h|\sin h}{h} = 0.$$

Therefore, we may express the derivative as

$$g'(x) = \begin{cases} |x|\cos x + \dfrac{x\sin x}{|x|}, & x \ne 0 \\ 0, & x = 0. \end{cases}$$

79.
$$\begin{aligned} \frac{dG}{dx} &= \frac{d}{dx}\sqrt{uv} = \frac{d}{dx}\sqrt{x(x+c)} \\ &= \frac{d}{dx}\sqrt{x^2+cx} = \frac{1}{2\sqrt{x^2+cx}}\frac{d}{dx}(x^2+cx) \\ &= \frac{2x+c}{2\sqrt{x^2+cx}} = \frac{x+(x+c)}{2\sqrt{x(x+c)}} \\ &= \frac{u+v}{2\sqrt{uv}} = \frac{A}{G} \end{aligned}$$